1. Let $X_1, X_2, \dots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \notin [1,3] \\ \theta 2^{-\theta}(t-1)^{\theta-1}, & \text{if } t \in [1,3] \end{cases}$$

Find a moment estimator for θ.

$$\bar{X} = \int_1^3 \theta 2^{-\theta} (t-1)^{\theta-1}\, t \, dt$$

u		dv
$t2^{-\theta}$		$\theta(t-1)^{\theta-1}$
	$+$	
$2^{-\theta}$		$(t-1)^{\theta}$
	$-$	
0		$\dfrac{(t-1)^{\theta+1}}{\theta+1}$

Note that the process of integration is different for $\theta = 1$, but the result is the same.

$$\left(t2^{-\theta}(t-1)^{\theta} - \frac{2^{-\theta}(t-1)^{\theta+1}}{\theta+1} \right) \Bigg|_1^3$$

$$= 3\,2^{-\theta}2^{\theta} - \frac{2^{-\theta}\,2^{\theta+1}}{\theta+1} = 3 - \frac{2}{\theta+1}$$

$$\Rightarrow \frac{2}{\theta+1} = 3 - \bar{X} \Rightarrow \theta + 1 = \frac{2}{3-\bar{X}} \Rightarrow \boxed{\hat{\theta} = \frac{2}{3-\bar{X}} - 1}$$