$$F_X(x) = \begin{cases} \frac{x+2}{3} & \text{if } x \in (-2,1) \\ 0 & \text{if } x \le -2 \\ 1 & \text{if } x \ge 1 \end{cases}$$

Quiz 2, Attempt 2

Find the cdf of $Y = X^2$ if $X \sim$ UNIF(-2, 1)

$$F_Y(y) = P(X^2 \le y) = 1\{y>0\}\, P(-\sqrt{y} \le X \le \sqrt{y})$$

$$= 1\{y>0\}\left(F_X(\sqrt{y}) - F_X(-\sqrt{y})\right)$$

$$= \begin{cases} \frac{\sqrt{y}}{3} + \frac{\sqrt{y}}{3} & y \in (0,1) \\ 1 - \left(\frac{-\sqrt{y}+2}{3}\right) & y \in [1,4) \\ 0 & y \le 0 \\ 1 & y \ge 4 \end{cases}$$

$$= \begin{cases} \frac{2\sqrt{y}}{3} & y \in (0,1) \\ \frac{1+\sqrt{y}}{3} & y \in [1,4) \\ 0 & y \le 0 \\ 1 & y \ge 4 \end{cases}$$

$$z = 1 - e^{-x} \qquad 1 - e^{-1}$$

$$e^{-x} = z - 1$$

Name:

$$-x = \ln(z-1)$$

$$x = -\ln(z-1)$$

Quiz 4, Attempt 1 (6.9c from the homework)

If X is UNIF(0,1) and Z = 1 - e^{-X}, use the transformation method to find the density function for Z.

$$f_Z(z) = f_X(x(z)) \left| \frac{d}{dz} x(z) \right| \underline{1}\{z \in B\}$$

$$= \underline{1}\{0 < z < 1 - \tfrac{1}{e}\} \left| \frac{-1}{z-1} \right|$$

$$= \boxed{\frac{\underline{1}\{0 < z < 1 - \frac{1}{e}\}}{1 - z}}$$

Check for integration to 1:

$$\int_0^{1-\frac{1}{e}} \frac{1}{1-z}\, dz = -\ln(1-z) \Big|_{z=0}^{z=1-e^{-1}} = -\ln(e^{-1}) + \ln(1)$$

$$= 1.$$