

$$\frac{1}{7} - \frac{1}{3} = \frac{3}{21} - \frac{7}{21} = -\frac{4}{21}$$

Name:

Quiz 3, Attempt 1

Find the density of the sum of two exponentially distributed random variables with means of 3 and 7.

$$f_X(x) = \frac{1}{3} e^{-x/3} \mathbb{1}\{x > 0\}$$

$$f_Y(y) = \frac{1}{7} e^{-y/7} \mathbb{1}\{y > 0\}$$

$$f_{X+Y}(x) = \int f_X(y) f_Y(x-y) dy$$

$$= \int \frac{1}{3} e^{-y/3} \mathbb{1}\{y > 0\} \frac{1}{7} e^{-\frac{x-y}{7}} \mathbb{1}\{x > y\} dy$$

$$= \mathbb{1}\{x > 0\} \int_0^x \frac{1}{21} e^{-y/3 - \frac{x}{7} + \frac{y}{7}} dy$$

$$= \mathbb{1}\{x > 0\} \frac{e^{-x/7}}{21} \int_0^x e^{-\frac{4y}{21}} dy$$

$$= \mathbb{1}\{x > 0\} \frac{e^{-x/7}}{21} \left(\frac{-21}{4} e^{-4y/21} \Big|_0^x \right)$$

$$= \mathbb{1}\{x > 0\} \frac{e^{-x/7}}{21} \frac{(-21)}{4} (e^{-4x/21} - 1)$$

$$= \mathbb{1}\{x > 0\} \frac{e^{-x/7}}{4} (1 - e^{-4x/21}) = \frac{\mathbb{1}\{x > 0\}}{4} (e^{-x/7} - e^{-x/3})$$