1. Let X be a random variable with density function

$$f(x) = \begin{cases} 1/2 & \text{if } x \in (-1,1) \\ 0 & \text{if } x \notin (-1,1). \end{cases}$$

   Compute the density function of $Y = (X-1)^2$.

2. Let X_1 and X_2 be independent identically distributed random variables with density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Compute the density function of (Y_1, Y_2), where $Y_1 = X_1/X_2$ and $Y_2 = X_1^2 X_2$.

3. Let X, Y be independent random variables with density functions

$$f(x) = \begin{cases} e^{-t} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0, \end{cases}$$

and

$$g(x) = \begin{cases} e^{t} & \text{if } x \leq 0 \\ 0 & \text{if } x > 0, \end{cases}$$

respectively. Compute the density of $X + Y$.

4. Let X_1, X_2, …, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of $X_{1:n}^n$.

5. Let X_1, X_2, …, X_n be independent identically distributed random variables with density function

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Approximate $P\left(\sum_{i=1}^{90} X_i \leq 77\right)$ in terms of $\Phi(\cdot)$, the cdf of a standard normal.

6. Let X_1 and X_2 be two independent random variables. X_1 is normal $N(1, 1)$ and X_2 is normal $N(0, 2)$. Compute

$$P\{X_1 + 2X_2 \leq 6\}$$

using one of the enclosed tables.

7. Let X_1, X_2, X_3 be independent identically distrubuted normal $N(0, 5)$ random variables. Compute

$$P\{X_1^2 + X_2^2 + X_3^2 \leq 15\}$$

using one of the enclosed tables.