1. Let X be a random variable with density function

$$f(x) = \begin{cases} 1/4 & \text{if } x \in (-2, 2) \\ 0 & \text{if } x \notin (-2, 2). \end{cases}$$

Compute the density function of $Y = X^3$. Use the cdf technique and the transformation method and compare the results (if both techniques are appropriate). Otherwise, state why one or both of the techniques are not appropriate.

2. Let X_1 and X_2 be independent identically distributed random variables with density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Compute the density function of (Y_1, Y_2), where $Y_1 = X_1/X_2$ and $Y_2 = X_1^2 X_2$.

3. Let X_1 and X_2 be independent identically distributed random variables with density function

$$f(x) = \begin{cases} \frac{e^x}{e-1} & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Compute the moment generating function of $2X_1 - 3X_2 + 2$.

4. Let X_1 and X_2 be independent random variables with density functions

$$f(x) = \frac{1}{2} e^{-|x|}$$

and

$$g(x) = \begin{cases} 1/2 & \text{if } x \in (-1, 1) \\ 0 & \text{if } x \notin (-1, 1), \end{cases}$$

respectively. Compute the density function of $X_1 + X_2$.

5. Let X_1, X_2, $\ldots$, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \frac{1}{1 + e^{-x}}.$$

Find the limiting distribution of $X_{1:n} + \log(n)$.

6. (a) If X is exponentially distributed with parameter 1, what transformation of X will yield a uniform(0,1) random variable?

   (b) If X is exponentially distributed with parameter 1, what transformation of X will yield a uniform(-1,1) random variable?

   (c) Does congergence in probability imply convergence in distribution? If no, provide a counter example.

7. (a) Does convergence in distribution imply convergence in probability? If no, provide a counter example.

   (b) State the law of large numbers including assumptions.

   (c) State the central limit theorem including assumptions.