1. Let X_1 and X_2 be independent identically distributed random variables with density function

$$f(x) = \begin{cases} \frac{e^x}{e-1} & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Compute the moment generating function of $2X_1 - 3X_2 + 2$.

2. Let X_1, X_2, …, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x^4 & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of $\frac{X_{n:n}}{n^{1/4}}$.

3. Let X_1, X_2, …, X_n be independent identically distributed random variables with density function

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Approximate $P\left(\sum_{i=1}^{90} X_i \leq 77\right)$ in terms of $\Phi(\cdot)$, the cdf of a standard normal.

4. Let $X_1, \ldots, X_{10}$ be independent random variables. The distribution of X_i is χ^2 with i degress of freedom. Find c such that

$$P\{X_1 + X_2 + X_3 + X_4 + X_5 \leq c(X_6 + X_7 + X_8 + X_9 + X_{10})\} = .95.$$

5. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } -\infty < t < \theta \\ e^{-(t-\theta)}, & \text{if } \theta \leq t < \infty. \end{cases}$$

Find a moment estimator for θ.

6. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$g(t, \theta) = \begin{cases} 0, & \text{if } -\infty < t < \theta \\ e^{-(t-\theta)}, & \text{if } \theta \leq t < \infty \end{cases}$$

Find the maximum likelihood estimator for θ and compute its bias.

7. Let X be a random variable with density function

$$f(t, \theta) = \frac{1}{(2\pi\theta)^{1/2}} e^{-t^2/(2\theta)}.$$

Compute the Cramer-Rao lower bound.

8. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with probaility mass function

$$f(t;\theta) = \frac{2!}{t!(2-t)!}\theta^t(1-\theta)^{2-t}, \quad \text{if} \;\; t = 0, 1, 2.$$

Find the uniformly minimum variance unbiased estimator for θ.

9. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$g(t,\theta) = \begin{cases} 0, & \text{if } -\infty < t < \theta \\ e^{-(t-\theta)}, & \text{if } \theta \leq t < \infty \end{cases}$$

Find the uniformly minimum variance unbiased estimator for θ (justify your answer).

10. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \frac{1}{2\theta} e^{-|t|/\theta}, \quad -\infty < t < \infty.$$

Find a minimal sufficient statistic for θ and show that it is complete.