1. Let $X$ be a random variable with density function

$$f(x) = \begin{cases} 2x & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Compute the moment generating function of $X$ and the moment generating function of $2X$.

2. Let $X_1$, $X_2$, …, $X_n$ be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x^2 & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of $\frac{X_{n:n}}{n^{1/2}}$.

3. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Approximate $P\left(\sum_{i=1}^{20} X_i \leq 12\right)$ in terms of $\Phi(\cdot)$, the cdf of a standard normal.

4. Let $X_1, X_2, X_3$ be independent normal random variables. $X_1$ is normal $N(0, 4)$, $X_2$ and $X_3$ are both normal $N(0, 9)$. Compute

$$P\{\frac{X_1}{(X_2^2 + X_3^2)^{1/2}} \leq .7\}$$

using one of the enclosed tables.

5. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \notin [1,3] \\ \theta 2^{-\theta}(t-1)^{\theta-1}, & \text{if } t \in [1,3] \end{cases}$$

Find a moment estimator for $\theta$.

6. Consider independent random samples $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ from normal distributions with a common mean, $\mu$, but with possibly different variances, $\sigma_1^2$ and $\sigma_2^2$, so that $X_i \sim N(\mu, \sigma_1^2)$ and $Y_i \sim N(\mu, \sigma_2^2)$. Assume the variances are known and find the MLE of $\mu$.

7. Let $X$ be a random variable with density function

$$h(t;\theta) = \frac{1}{2\theta}e^{-|t|/\theta}, \quad -\infty < t < \infty.$$

Compute the Cramer-Rao lower bound.

8. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with probaility mass function

$$f(t;\theta) = \frac{2!}{t!(2-t)!}\theta^t(1-\theta)^{2-t}, \quad \text{if } \; t = 0, 1, 2.$$

Find the uniformly minimum variance unbiased estimator for $\theta$.

9. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with $P\{X_1 = 1\} = \theta$ and $P\{X_1 = 0\} = 1-\theta$ for some $\theta \in (0,1)$. Find the uniformly minimum variance unbiased estimator for $\theta(1-\theta)$.

10. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \leq 1 \\ \theta t^{-\theta-1}, & \text{if } t \geq 1 \end{cases}$$

Find a sufficient statistic for $\theta$.