1. Let X be a random variable with density function

$$f(x) = \frac{1}{2}e^{-|x|}.$$

Compute the moment generating function of X and use it to compute $E(X^2)$.

2. Let X_1, X_2, …, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of $X_{n:n}/n$.

3. Let X_1, X_2, ..., X_n be independent identically distributed random variables with density function

$$f(x) = \begin{cases} 1/2 & \text{if } x \in (-1, 1) \\ 0 & \text{if } x \notin (-1, 1). \end{cases}$$

Approximate $P\left(\sum_{i=1}^{100} X_i \leq 0\right)$ in terms of $\Phi(\cdot)$, the cdf of a standard normal.

4. Let X_1, X_2, X_3 be independent identically distrubuted normal $N(0,5)$ random variables. Compute c such that

$$P\{X_1^2 + X_2^2 + X_3^2 \leq c\} = .1$$

using one of the enclosed tables.

5. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$f(t) = \begin{cases} 0, & \text{if } t < \theta \\ e^{-(t-\theta)}, & \text{if } t > \theta. \end{cases}$$

Find a moment estimator for θ, where θ is the unknown parameter.

6. Consider a random sample of size n from a distribution with pdf $f(x;\theta) = \frac{1}{\theta}$ if $0 < x \leq \theta$, and zero otherwise; $0 < \theta$. Compute the MLE and MME for θ. Compare the bias and MSE of the two estimators.

7. Let X be a random variable with density function

$$h(t;\theta) = \frac{1}{2\theta} e^{-|t|/\theta}, \quad -\infty < t < \infty.$$

Compute the Cramer-Rao lower bound.

8. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with probaility mass function

$$f(t;\theta) = \theta(1-\theta)^{t-1}, \ \text{ if } t = 1, 2, \ldots.$$

Find the uniformly minimum variance estimator for

$$\tau = \frac{1}{\theta}$$

.

9. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed Poisson θ random variables. Find the uniformly minimum variance unbiased estimator for $\tau = \tau(\theta) = e^{-\theta}$.

10. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \notin [0,\theta] \\ \dfrac{2t}{\theta^2}, & \text{if } t \in [0,\theta]. \end{cases}$$

Find a sufficient statistic for θ.