

1. Let  $X$  be a random variable with density function

$$f(x) = \begin{cases} 1/4 & \text{if } x \in (-2, 2) \\ 0 & \text{if } x \notin (-2, 2). \end{cases}$$

Compute the density function of  $Y = (X - 1)^2$ .

2. Let  $X_1$  and  $X_2$  be independent identically distributed random variables with density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Compute the density function of  $(Y_1, Y_2)$ , where  $Y_1 = X_1$  and  $Y_2 = X_1 + 2X_2$ .

3. Let  $X_1$  and  $X_2$  be independent identically distributed random variables with density function

$$f(x) = \begin{cases} \frac{e^x}{e-1} & \text{if } x \in (0, 1) \\ 0 & \text{if } x \notin (0, 1). \end{cases}$$

Compute the moment generating function of  $2X_1 - 3X_2 + 2$ .

4. Let  $X_1$  and  $X_2$  be independent random variables with density functions

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

and

$$g(x) = \begin{cases} 1 & \text{if } x \in (0, 1) \\ 0 & \text{if } x \notin (0, 1), \end{cases}$$

respectively. Compute the density function of  $X_1 + X_2$ .

5. Let  $X_1, X_2, \dots, X_n$  be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of  $X_{n:n}/n$ .

6. Let  $X_1, X_2, \dots, X_n$  be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x^4 & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of  $\frac{X_{n:n}}{n^{1/4}}$ .

7. Let  $X_1, X_2, \dots, X_n$  be independent identically distributed random variables with density function

$$f(x) = \begin{cases} 1 & \text{if } x \in (0, 1) \\ 0 & \text{if } x \notin (0, 1). \end{cases}$$

Approximate  $P(\sum_{i=1}^{90} X_i \leq 77)$  in terms of  $\Phi(\cdot)$ , the cdf of a standard normal.