1. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with probaility mass function

$$f(t;\theta) = \frac{2!}{t!(2-t)!}\theta^t(1-\theta)^{2-t}, \text{ if } t = 0, 1, 2.$$

Find the uniformly minimum variance unbiased estimator for θ.

$X_i \sim iid$ BIN$(2,\theta)$.

Let $r(\underline{X}) = \mathbb{1}\{X_i \in \{0,1,2\}$ for $i=1,2,3\}$.

$$f(\underline{X};\theta) = \prod r(\underline{x})\binom{2}{X_i}\theta(1-\theta)^{2-X_i} = \left[\prod \binom{2}{X_i}\right]\theta^n(1-\theta)^{2n-\Sigma X_i} r(\underline{X})$$

Thus ΣX_i is sufficient for θ by the factorization criterion.

$E(\overline{X}) = E(X_1) = 2\theta$, so $E\left(\frac{\overline{X}}{2}\right) = \theta$.

Since BIN$(2,\theta)$ is a member of the REC, ΣX_i is complete.

$\frac{\overline{X}}{2}$ is an unbiased function of a sufficient & complete statistic, so it is the UMVUE for θ.