MATH 5010 – Exam 2

Name:

Date:

You will notice that the questions on this exam resemble questions that you have seen before. However, many of them are NOT quite the same. Read each question carefully. Each question is worth 3 points. [redacted]. *Although partial credit is not guaranteed, some partial credit may be awarded. Therefore it may be in your best interest to write your steps down neatly and carefully.*

1. I have a p-coin. Each toss of the coin is independent of other tosses, and each toss results in HEADS with probability p. I will toss this coin until it lands on heads. Let X be the number of times it will land on TAILS. Find the mass function of X.

$$p(x) = \begin{cases} (1-p)^x p & x = 0, 1, 2, \ldots \\ 0 & \text{o/w.} \end{cases}$$

2. Suppose X has c.d.f. given by

$$F(x) = \begin{cases} x^2 & x \in (0,1), \\ 1 & x \geq 1, \\ 0 & x \leq 0. \end{cases}$$

Find the variance of X.

$$f_X = \begin{cases} 2x & x \in (0,1), \\ 0 & \text{else} \end{cases}$$

$$E(X) = \int_0^1 x\, 2x\, dx = \int_0^1 2x^2\, dx = \frac{2x^3}{3}\Big|_0^1 = \frac{2}{3}$$

$$E(X^2) = \int_0^1 x^2\, 2x\, dx = \int_0^1 2x^3\, dx = \frac{2x^4}{4}\Big|_0^1 = \frac{1}{2}$$

$$\mathrm{Var}\, X = E(X^2) - E(X)^2 = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{2} - \frac{4}{9}$$

$$= \frac{9-8}{18} = \boxed{\frac{1}{18}}$$

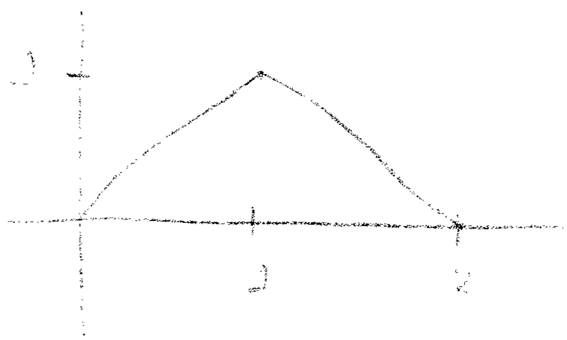
3. Suppose X and Y have joint density given by

$$f(x,y) = \begin{cases} 3e^{-x-3y} & x, y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the expected value of X.

$E(X) = 1$

4. Suppose X and Y are independent and uniformly distributed on $(0, 1)$. Find the density of $W = X + Y$.



$$f(w) = \begin{cases} 0 & w < 0, \\ w & w \in (0,1), \\ 2 - w & w \in (1,2) \\ 0 & w > 2. \end{cases}$$

5. Suppose X and Y are independent. Suppose also that X is uniform on $(0,1)$ and Y is exponentially distributed with parameter 1. Let $U = X + 2Y$ and $V = -X/3 + Y$. Find the joint density of U and V.

$$f_{XY}(x,y) = \begin{cases} e^{-y} & x \in (0,1) \quad y > 0, \\ 0 & \text{o/w.} \end{cases}$$

$u = x + 2y$

$v = \frac{-x}{3} + y$

$$J = \begin{vmatrix} 1 & 2 \\ \frac{-1}{3} & 1 \end{vmatrix} = -\frac{2}{3} - 1 = \frac{-5}{3}$$

$x = \frac{3}{5}(u - 2v)$

$y = \frac{3}{5}v + \frac{1}{5}u$

$$f_{UV}(u,v) = f_{XY}\left(\frac{3}{5}(u-2v), \frac{3}{5}v + \frac{1}{5}u\right)\frac{3}{5}$$

$$= \begin{cases} \exp\left[-\left(\frac{3}{5}v + \frac{1}{5}u\right)\right]\frac{3}{5} & \frac{3}{5}(u-2v) \in (0,1) \text{ and } \left(\frac{3}{5}v + \frac{1}{5}u\right) > 0 \\ 0 & \text{o/w} \end{cases}$$

6. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .2, and his second will lead independently to a sale with probability .4. Any sale made is equally likely to be either for the deluxe model, which costs \$500, or the standard model, which costs \$300. Let X be the total sales in dollars. Find the Expected value and variance of X.

7. The width of a slot of a duralumin forging is (in inches) normally distributed with $\mu = .9$ and $\sigma = .003$ The specification limits were given as $.9 \pm .006$. What percentage of forgings will be within specification? Express your answer in terms of $\Phi(x)$, the cdf of a standard normal.

$X \sim N(.9, (.003)^2)$

$P\left(\frac{-.006}{.003} < \frac{X-.9}{.003} < \frac{.006}{.003}\right)$

$\Phi(2) - \Phi(-2)$

$= 1 - 2\Phi(-2)$

8. Jay has two jobs to do, one after the other. Each attempt at job i takes one hour and is successful with probability p_i. If $p_1 = .2$ and $p_2 = .4$, what is the probability that it will take Jay more than 4 hours to be successful on both jobs?

$$p(x,y) = \begin{cases} (.8)^{x-1}(.2)(.6)^{y-1}(.4) & x = 1,2,3, \\ & y = 1,2,3,\ldots \\ 0 & \text{o/w.} \end{cases}$$

$$P(X+Y > 4) = 1 - P(X+Y \le 4)$$
$$= 1 - \sum_{x=1}^{3} \sum_{y=1}^{4-x} (.8)^{x-1}(.2)(.6)^{y-1}(.4)$$