

## MATH 5010 – Exam 2

Name:

Date:

*You may notice that some questions on this exam resemble questions that you have seen before. However, many of them are NOT quite the same. Read each question carefully. Each question is worth 4 points; your total exam score will be either 25 or the sum of your individual points, whichever is less. Although partial credit is not guaranteed, some partial credit may be awarded. Therefore it may be in your best interest to write your steps down neatly and carefully.*

1. I have a  $p$ -coin. Each toss of the coin is independent of other tosses, and each toss results in HEADS with probability  $p$ . I will toss this coin until it lands on heads 5 times. Let  $X$  be the number of times it will land on TAILS. Find the mass function of  $X$ .

2. Find the fourth moment,  $E(X^4)$ , of an exponential random variable with parameter 1 (i.e. EXP(1)).

3. Suppose  $X$  and  $Y$  have joint density, which, for some value of  $C$ , is given by

$$f(x, y) = \begin{cases} Ce^{-x-3y} & x, y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the variance of  $Y$ .

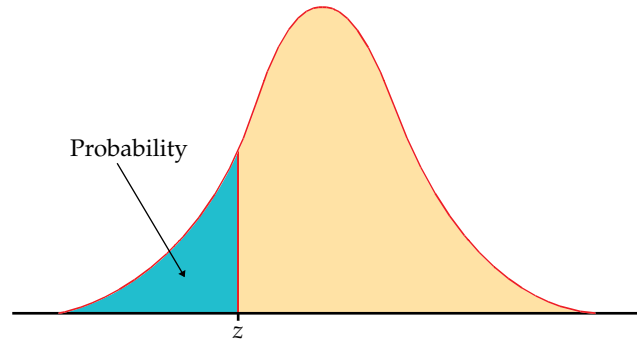
4. Suppose  $X$  and  $Y$  are i.i.d UNIF(-1,1). Find the density of  $W = X + Y$ .

5. Suppose  $X$  and  $Y$  are independent. Suppose also that  $X$  is uniform on  $(0, 1)$  and  $Y$  is exponentially distributed with parameter 1. Let  $U = 2X + 4Y$  and  $V = -X/3 + Y$ . Find the joint density of  $U$  and  $V$ .

6. The width of a slot of a duralumin forging is (in inches) normally distributed with a mean of .7 and a variance of .0004. The specification limits were given as  $.8 \pm .005$ . What percentage of forgings will be within specification? Express your answer in terms of  $\Phi(x)$ , the cdf of a standard normal, and then use the table at the back of this exam to evaluate it.

7. Jay has two jobs to do, one after the other. Each attempt at job  $i$  takes one hour and is successful with probability  $p_i$ , independent of previous attempts. If  $p_1 = .3$  and  $p_2 = .5$ , what is the probability that it will take Jay more than 5 hours to be successful on both jobs?

Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

**TABLE A****Standard normal probabilities**

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



