MATH 5010 – Exam 1

Name:

Date:

You may notice that some questions on this exam resemble questions that you have seen before. However, many of them are NOT quite the same. Read each question carefully. Each question is worth 3 points; your total exam score will be either 25 or the sum of your individual points, whichever is less. Although partial credit is not guaranteed, some partial credit may be awarded. Therefore it may be in your best interest to write your steps down neatly and carefully.

1. Bad drivers crash in a given year with probability 0.4 while a safe driver will crash with probability 0.2. An insurance company receives an application from a person that was in an accident last year and wants to charge an appropriate amount depending on whether the applicant is a good driver. It is estimated that 40% of the applicants are good drivers. What is the probability that this particular applicant is a good driver?

$A = \{\text{Safe driver}\}$

$B = \{\text{Crashed last year}\}$

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)} = \frac{(0.2)(0.4)}{\mathbb{P}(B \mid A)\mathbb{P}(A) + \mathbb{P}(B \mid A^c)\mathbb{P}(A^c)}$$

$$= \frac{(0.2)(0.4)}{(0.2)(0.4) + (0.4)(0.6)} = \frac{1}{4}$$

2. Consider a 2-dimensional integer lattice. Suppose that a robot starting at (0,0) must pick up a load at (27,5) and deliver it to (45,17). In a single move, the robot can either go up by one (eg (0,0) to (0,1)) or go right by one (eg (0,0) to (1,0)). The robot chooses a path at random so that each possible path is equally likely. What is the probability that the robot will hit the landmine at (40,10)?

$$\frac{\binom{32}{27}\binom{18}{13}\binom{12}{5}}{\binom{32}{27}\binom{30}{18}}$$

3. Suppose n smokers commit to quit smoking for 90 days. Each smoker will quit for the 90 day period with probability p, independently of others. What is the probability that fewer than m of the n smokers succeed? Note that $0 \leq m \leq n$ are integers.

$$\sum_{x=0}^{m-1} \binom{n}{x} p^x (1-p)^{n-x}$$

4. 12 Vulcans walk into a bar on the 456th day of the Vulcan year; it's the last day of the year. They look around and notice that 3 other Vulcans are there with them. What is the probability that there are at least two vulcans in the bar with the same birthday.

$$1 - \frac{456 \cdot 455 \cdot \ldots \cdot 442}{(456)^{15}}$$

5. The Jones family and the Smith family each have two children. Assume that each child born is male with probability 0.5 independently of other births. The oldest child in the Jones family is a girl and at least one of the children in the Smith family is a girl. What is the probability that both children in both families are girls?

$A = \{\text{Jones kids are both girls}\}$

$B = \{\text{Smith kids are both girls}\}$

$$\mathbb{P}(A) = \frac{N(\{GG\})}{N(\{GG, GB\})} = \frac{1}{2}$$

$$\mathbb{P}(B) = \frac{N(\{GG\})}{N(\{GG, GB, BG\})} = \frac{1}{3}$$

$$\mathbb{P}(A \cap B) = \frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{6}$$

6. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the prostate-specific antigen (PSA) that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, increasing to approximately .268 if the man does have cancer. If, on the basis of other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that the test indicated an elevated PSA level?

$C = \{\text{has cancer}\}$

$P = \{\text{elevated PSA}\}$

$\mathbb{P}(P \mid C^c) = 0.135$

$\mathbb{P}(P \mid C) = 0.268$

$\mathbb{P}(C) = 0.70$

$$\mathbb{P}(C \mid P) = \frac{\mathbb{P}(P \mid C)\,\mathbb{P}(C)}{\mathbb{P}(P)} = \frac{\mathbb{P}(P \mid C)\,\mathbb{P}(C)}{\mathbb{P}(P \mid C)\,\mathbb{P}(C) + \mathbb{P}(P \mid C^c)\,\mathbb{P}(C^c)}$$

$$= \frac{0.268\,(0.7)}{0.268\,(0.7) + 0.135\,(0.3)}$$

$$\approx 0.82$$

7. If 8 identical blackboards are to be divided among 4 schools, how many divisions are possible?

$$\binom{11}{3}$$

8. A group of 6 men and 6 women is randomly divided into two groups. What is the probability that one group will have more men in it than the other?

$$1 - \frac{\binom{6}{3}\binom{6}{3}}{\binom{12}{6}}$$

9. A certain coin, when tossed, will land on heads with probability 1/3. If I toss this coin 12 times, what is the probability that it lands on heads exactly 4 times?

$$\binom{12}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^8$$