MATH 3080 – Exam 1

Name:

Date:

You will notice that the questions on this exam resemble questions that you have seen before. However, many of them are NOT quite the same. Read each question carefully. Each question is worth 3 points for a total of 27 points. Although partial credit is not guaranteed, some partial credit may be awarded. Therefore it may be in your best interest to write your steps down neatly and carefully.

1. From a group of 11 women and 15 men, a committee consisting of 4 women and 3 men is to be chosen. How many different committees are possible if three of the men hate each other and refuse to work with any of the other three?

$$\binom{11}{4}\left[\binom{15}{3} - \binom{3}{2}\binom{12}{1} - \binom{3}{3}\binom{12}{0}\right] = 137{,}940$$

2. Consider a 2-dimensional integer lattice. Suppose that a robot starting at (0,0) must pick up a load at (27,5) and deliver it to (45,17). In a single move, the robot can either go up by one (eg (0,0) to (0,1)) or go ~~over~~ right by one (eg (0,0) to (1,0)). How many different paths could the robot take?

$$\binom{32}{5}\binom{30}{12} = \binom{32}{27}\binom{30}{18} = \binom{32}{5}\binom{30}{18} = \binom{32}{27}\binom{30}{12}$$

$$\approx 1.74 * 10^{13}$$

3. If 9 new teachers are to be divided among 3 schools, how many divisions are possible?

$3^9 = 19683$

4. A pair of dice is rolled until a sum of 5 or 11 is obtained and then the experiment stops. What is the probability that a sum of 5 is achieved on the 27th roll?

	1	2	3	4	5	6
1				5		
2			5			
3		5				
4	5					
5						11
6					11	

$$\frac{30^{26}\,4}{36^{27}} \simeq .0009706$$

5. A small community organization consists of 25 families, of which 4 have one child, 8 have two children, 5 have three children, 5 have four children, and 3 has five children. If one of the children is selected at random, what is the probability that the child has exactly 2 siblings?

$$\# \text{ of children} = 4(1) + 8(2) + 5(3) + 5(4) + 3(5)$$

$$= 70$$

$$\# \text{ of children with exactly 2 siblings} = 5(3) = 15$$

$$\mathbb{P}(A) = \frac{N(A)}{N(S)} = \frac{15}{70} \approx .2143$$

6. A bowl contains 15 white balls and 27 black balls. Three balls are randomly scooped out. What is the probability that at least one black and at least one white ball are scooped out?

Let $A = \{\text{at least one of each color}\}$

$B = \{\text{all black}\}$

$W = \{\text{all white}\}$

$$\mathbb{P}(A) = 1 - \mathbb{P}(A^c) = 1 - \mathbb{P}(B \cup W) = 1 - \mathbb{P}(B) - \mathbb{P}(W)$$

$$= 1 - \frac{27 \cdot 26 \cdot 25}{42 \cdot 41 \cdot 40} - \frac{15 \cdot 14 \cdot 13}{42 \cdot 41 \cdot 40}$$

$$\approx .7056$$

7. A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random. He flips the coin 5 times. Each toss results in a heads. What is the probability that it is the fair coin?

$A = \{\text{heads all five tosses}\}$

$F = \{\text{Fair coin}\}$

$$\mathbb{P}(F \mid A) = \frac{\mathbb{P}(A \mid F)\,\mathbb{P}(F)}{\mathbb{P}(A)} = \frac{\mathbb{P}(A \mid F)\,\mathbb{P}(F)}{\mathbb{P}(A \mid F)\,\mathbb{P}(F) + \mathbb{P}(A \mid F^c)\,\mathbb{P}(F^c)}$$

$$= \frac{\left(\frac{1}{2}\right)^5 \frac{1}{2}}{\left(\frac{1}{2}\right)^6 + \frac{1}{2}}$$

$$= \frac{\frac{1}{64}}{\frac{1}{64} + \frac{32}{64}} = \boxed{\frac{1}{33}}$$

8. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the prostate-specific antigen (PSA) that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, increasing to approximately .268 if the man does have cancer. If, on the basis of other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that the test indicated an elevated PSA level?

$E = \{\text{elevated PSA}\}$

$C = \{\text{has cancer}\}$

$$\mathbb{P}(C \mid E) = \frac{\mathbb{P}(E \mid C)\mathbb{P}(C)}{\mathbb{P}(E)} = \frac{\mathbb{P}(E \mid C)\mathbb{P}(C)}{\mathbb{P}(E \mid C)\mathbb{P}(C) + \mathbb{P}(E \mid C^c)\mathbb{P}(C^c)}$$

$$= \frac{(.268)(.70)}{(.268)(.70) + (.135)(.30)}$$

$$\approx .8224$$

9. A certain coin, when tossed, will land on heads with probability 1/3. If I toss this coin 12 times, what is the probability that it lands on heads exactly 4 times?

$$\binom{12}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^8 = \binom{12}{8}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^8 \approx .2384$$