

MATH 5010 – Final Exam

Name:

Date:

You will notice that the questions on this exam resemble questions that you have seen before. However, many of them are NOT quite the same. Read each question carefully. Each question is worth 3 points for a total of 27 points. Although partial credit is not guaranteed, some partial credit may be awarded. Therefore it may be in your best interest to write your steps down neatly and carefully.

1. When playing a certain board game, 5 dice are rolled at the same time. Let X be the sum of the numbers on all five dice. Find the expected value of X .

2. Suppose X has c.d.f. given by

$$F(x) = \begin{cases} x & x \in (0, 1), \\ 1 & x \geq 1, \\ 0 & x \leq 0. \end{cases}$$

Find the variance of X .

3. Suppose X and Y are independent. Suppose also that X is uniform on $(0, 1)$ and Y is exponentially distributed with parameter 1. Let $U = 2X + Y$ and $V = -X/5 + Y$. Find the joint density of U and V .

4. The width of a slot of a duralumin forging is (in inches) normally distributed with $\mu = .9$ and $\sigma = .003$. The specification limits were given as $.9 \pm .009$. What percentage of forgings will be within specification? Express your answer in terms of $\Phi(x)$, the cdf of a standard normal.

5. Consider a 2-dimensional integer lattice. Suppose that a robot starting at $(0,0)$ must pick up a load at $(26,5)$ and deliver it to $(45,20)$. In a single move, the robot can either go up by one (eg $(0,0)$ to $(0,1)$) or go right by one (eg $(0,0)$ to $(1,0)$). How many different paths could the robot take?

6. A pair of dice is rolled until a sum of 5 or 12 is obtained and then the experiment stops. What is the probability that a sum of 5 is achieved on the 27th roll?

7. A bowl contains 10 white balls and 33 black balls. Three balls are randomly scooped out. What is the probability that at least one black and at least one white ball are scooped out?

8. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the prostate-specific antigen (PSA) that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, increasing to approximately .268 if the man does have cancer. If, on the basis of other factors, a physician is 80 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that the test indicated an elevated PSA level?

9. The random variables X and Y have a joint density function given by

$$f(x, y) = \begin{cases} \frac{2}{x}e^{-2x} & 0 < y < x, \\ 0 & \text{otherwise.} \end{cases}$$

Find the expected value of Y .

10. Student scores on exams given by a certain instructor have mean 74 and standard deviation 14. This instructor is about to give two exams, one to a class of size 25 and the other to a class of size 64. Approximate the probability that the average score in the class of size 25 exceeds the other class's average by more than 2 points. Express your answer in terms of $\Phi(x)$.

11. An insurance company has 10,000 automobile policyholders. The expected yearly claim per policyholder is \$240, with a standard deviation of \$800. Approximate the probability that the total yearly claim exceeds \$3 million. Express your answer in terms of $\Phi(x)$.

12. In a workshop there are 4 kinds of beds, 3 kinds of closets, 2 kinds of shelves and 7 kinds of chairs. In how many ways can a person decorate his room if he wants to buy from the workshop one shelf, one bed and one of the following: a chair or a closet?