16. How many subsets of size 4 of the set $S = \{1, 2, \ldots, 20\}$ contain at least one of the elements $1, 2, 3, 4, 5$?

20. Balls are randomly removed from an urn initially containing 20 red and 10 blue balls. What is the probability that all of the red balls are removed before all of the blue ones have been removed?

3.15. In a certain species of rats, black dominates over brown. Suppose that a black rat with two black parents has a brown sibling.

    (a) What is the probability that this rat is a pure black rat (as opposed to being a hybrid with one black and one brown gene)?

    (b) Suppose that when the black rat is mated with a brown rat, all 5 of their offspring are black. Now what is the probability that the rat is a pure black rat?

Background that may help: Each rat has two color-determining genes—Bb, BB, or bb. Let B be the black gene and b be the brown gene. When two rats mate, they each contribute one of their genes to the offspring. So if Dad has BB and Mom has Bb, then both parents are black. Dad will contribute a B gene to the offspring (that's all he has to give) and Mom will contribute either a B or a b, with probability 0.5 each. So the offspring will be Bb or BB. That is, the offspring will be black, either pure black or a hybrid.

4.3. A coin that, when flipped, comes up heads with probability p is flipped until either heads or tails has occurred twice. Find the expected number of flips.

5.18. There are two types of batteries in a bin. When in use, type i batteries last (in hours) an exponentially distributed time with rate $\lambda_i, i = 1, 2$. A battery that is randomly chosen from the bin will be a type i battery with probability $p_i, \sum_{i=1}^{2} p_i = 1$. If a randomly chosen battery is still operating after t hours of use, what is the probability that it will still be operating after an additional s hours?

Mark each of the following questions true or false: (1 point for every two you get right)

The cumulative distribution function of a continuous random variable is defined for all real numbers and differentiable everywhere except at (at most) one point.

The cumulative distribution function of a discrete random variable is continuous at most places.

A random variable whose set of possible outcomes is finite is necessarily discrete.

A random variable whose set of possible outcomes is infinite is necessarily continuous.

A set of events can be independent without being pairwise independent.

When p is large, a BIN(n,p) random variable can be approximated by an normal random variable with the appropriate mean and variance.