MATH 5010 – Quiz 12

Name:

Date:

5.40 If X is uniformly distributed over $(-1, 1)$, find the variance of $Y = e^X$.

$$F_Y(y) = \mathbb{P}(Y \le y) = \mathbb{P}(e^X \le y) = \mathbb{P}(X \le \log(y)) = F_X(\log(y))$$

$$f_Y(y) = f_X(\log(y))\frac{1}{y} = \begin{cases} \frac{1}{2y} & \frac{1}{e} < y < e, \\ 0 & \text{o/w.} \end{cases}$$

$$E(Y) = \int_{e^{-1}}^{e} \frac{1}{2}\, dy = \frac{1}{2}\left(e - \frac{1}{e}\right)$$

$$E(Y^2) = \int_{e^{-1}}^{e} y/2\, dy = \frac{y^2}{4}\Big|_{e^{-1}}^{e} = \frac{e^2}{4} - \frac{e^{-2}}{4}$$

$$\text{var}(Y) = \left(\frac{e^2}{4} - \frac{e^{-2}}{4}\right) - \frac{1}{4}\left(e - \frac{1}{e}\right)^2$$

$$= \cancel{\frac{e^2}{4}} - \frac{e^{-2}}{4} - \cancel{\frac{e^2}{4}} - \frac{e^{-2}}{4} + \frac{2}{4}$$

$$= \frac{1}{2} - \frac{1}{2e^2} = \boxed{\frac{1}{2}\left(1 - \frac{1}{e^2}\right)}$$

Note: Alternatively, you could solve this using the fact that $E[g(X)] = \int_{\mathbb{R}} g(x) f(x)\, dx$.