

MATH 5010 – Quiz

Name:

Find the third moment, $E(X^3)$, of a Poisson random variable, X .

Note that $E(X) = \text{var}(X) = \lambda$.

Therefore $\lambda = E(X^2) - \lambda^2$ and $E(X^2) = \lambda^2 + \lambda$.

$$E(X^3) = \sum_{x=0}^{\infty} \frac{x^3 e^{-\lambda} \lambda^x}{x!} = \sum_{x=1}^{\infty} \frac{x^3 e^{-\lambda} \lambda^x}{x!} = \sum_{x=1}^{\infty} \frac{x^2 e^{-\lambda} \lambda^x}{(x-1)!} = \sum_{x-1=0}^{\infty} \frac{x^2 e^{-\lambda} \lambda \lambda^{x-1}}{(x-1)!}$$

With the change of variables $y = x - 1$, this becomes

$$\begin{aligned} \lambda \sum_{y=0}^{\infty} \frac{(y+1)^2 e^{-\lambda} \lambda^y}{y!} &= \lambda \sum_{y=0}^{\infty} \frac{(y^2 + 2y + 1) e^{-\lambda} \lambda^y}{y!} = \lambda \left[(\lambda^2 + \lambda) + 2(\lambda) + 1 \right] \\ &= \lambda^3 + 3\lambda^2 + \lambda \end{aligned}$$