MATH 5010 - Quiz

Name:
3.90 Suppose that $n$ independent trials, each of which results in either 0 , 1 , or 2 , with respective probabilities $p_{0}, p_{1}$, and $p_{2}$, are performed. Find the probability that outcomes 1 and 2 both occur at least once.

$$
\text { Let } \begin{aligned}
A & =\{1 \text { occurs at least once }\}, \\
B & =\{2 \text { occurs at least once }\} .
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{P}(A \cap B) & =1-\mathbb{P}\left(A^{c} \cup B^{c}\right) \\
& =1-\left[\mathbb{P}\left(A^{c}\right)+\mathbb{P}\left(B^{c}\right)-\mathbb{P}\left(A^{c} \cap B^{c}\right)\right] \\
& =1-\left[\left(1-p_{1}\right)^{n}+\left(1-p_{2}\right)^{n}-p_{0}^{n}\right] \\
& =1-\left(1-p_{1}\right)^{n}-\left(1-p_{2}\right)^{n}+p_{0}^{n}
\end{aligned}
$$

