

MATH 5010 – Quiz 7

Name:

Date:

3.37 A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random.

1. When he flips it, it shows heads. What is the probability that it is the fair coin?

$$\text{Let } F = \{\text{fair coin is used}\}, \\ H = \{\text{Heads}\}$$

$$P(F|H) = \frac{P(F \cap H)}{P(H)} = \frac{P(H|F)P(F)}{P(H)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{P(H|F)P(F) + P(H|F^c)P(F^c)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \boxed{\frac{1}{3}}$$

2. When he flips the coin a second time, it shows heads again. Now what is the probability that it is the fair coin?

$$P(F|HH) = \frac{P(HH|F)P(F)}{P(HH)} = \frac{P(HH|F)P(F)}{P(HH|F)P(F) + P(HH|F^c)P(F^c)}$$

$$= \frac{\left(\frac{1}{2}\right)^2 \cdot \frac{1}{2}}{\frac{1}{8} + \frac{1}{2}} = \frac{\frac{1}{8}}{\frac{1}{8} + \frac{1}{2}} = \boxed{\frac{1}{5}}$$

3. When he flips the coin a third time, it shows heads again. Now what is the probability that it is the fair coin?

$$P(F|HHH) = \frac{P(HHH|F)P(F)}{P(HHH|F)P(F) + P(HHH|F^c)P(F^c)} = \frac{\left(\frac{1}{2}\right)^3}{\frac{1}{16} + \frac{1}{2}}$$

$$= \boxed{\frac{1}{9}}$$