**Chapter #5 – Continuous Random Variables**

**Question #1:** Let X be a random variable with the following probability density function . (a) What is the value of the constant c in the PDF given by ? (b) What is the cumulative distribution function (CDF) of X?

1. To find constant, we have that . Since we have found that , it follows that the constant .
2. We have that . We can express the CDF as .

**Question #4:** The probability density function of X, the lifetime of a certain type of electronic device (measured in hours) is given by . (a) Calculate the . (b) What is the cumulative distribution function (CDF) of X? (c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours?

1. We find as follows .
2. We have that . We can express the CDF as .
3. Let be the event that a single device works for at least 15 hours. We thus have that . If we let denote the event that at least three will work for at least 15 hours, then we have .

**Question #7:** The density function of X is given by . (a) If we are given that the expected value is , find the constants and . (b) Find the CDF.

1. We will create a system of two equations in terms of and so we can solve for them. The first is obtained from the fact that we know . Thus, we have that . The second equation is found from the fact the probability density function over its domain must sum to , implying that we have . These two equations allow us to solve for and .
2. We calculate that . Thus, we have that .

**Question #10:** Trains headed for destination A arrive at the train station at 15-minute intervals starting at 7 AM, whereas trains headed for destination B arrive at 15-minute intervals starting at 7:05 AM. (a) If a certain passenger arrives at the station at a time uniformly distributed between 7 and 8 AM and then gets on the first train that arrives, what proportion of time does he or she go to destination A? (b) What if the passenger arrives at a time uniformly distributed between 7:10 and 8:10 AM?

1. . Here, we calculated each probability using the CDF of , which is given by the formula for all . Note that the CDF of is equal to zero for every and is equal to 1 for every . (Draw clock to get four intervals.)
2. The answer is identical to the one given above.

**Question #13:** You arrive at a bus stop at 10 o’clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. (a) What is the probability that you will have to wait longer than 10 minutes? (b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

1. If we let be the random variable equal to the arrival time of the bus, then we have that . Thus, .
2. .

**Question #16:** The annual rainfall (in inches) in a certain region is normally distributed with and . What is the probability that, starting with this year, it will take over 10 years before a year occurs having a rainfall of over 50 inches?

* If we let be the random variable equal to the annual rainfall in the region, then we have that . The probability that rainfall is over 50 inches in any year is . If we let denote the event that the rainfall is less than 50 in ten consecutive years, then we see that .

**Question #19:** Let X be a normal random variable with mean 12 and variance 4. Find the value of such that .

* We have that , so . From the z-table, we have that , which thus implies that , so we must solve the equation , which gives .

**Question #22:** The width of a slot of a duralumin forging is (in inches) normally distributed with and . The specification limits were . (a) What percentage of forgings will be defective? (b) What is the maximum value of that will permit no more than 1 in 100 defectives when the widths are normally distributed with and ?

1. We have that . Thus, are defective.
2. We start with when . Thus, we must solve the equation , which gives .

**Question #31:** (a) A fire station is to be located along a road of length . If fires occur at points uniformly chosen on , where should the station be located so as to minimize the expected distance from the fire? That is, choose so as to minimize when X is uniformly distributed over . (b) Now suppose that the road is of infinite length —stretching from point 0 outward to . If the distance of a fire from point 0 is exponentially distributed with rate , where should the fire station now be located? That is, we want to minimize , where X is now exponential with rate .

1. We have that . Since we want to minimize this, we take the derivative and set equal to zero: . Thus, we can minimize the expected value by choosing the midpoint of the interval .
2. Using integration by parts we can find that . After differentiating and setting equal to zero, we discover that , which gives the minimum value at .

**Question #34:** Jones figures that the total number of thousands of miles that an auto can be driven before it would need to be junked is an exponential random variable with parameter . Smith has a used car that he claims has been driven only 10,000 miles. a) If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it? b) Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed, but is (thousands of miles) uniformly distributed over .

1. If is the continuous random variable equal to the total number of thousands of miles that a car can be driven before it must be junked, then we know . The PDF of is while the CDF of is . We also know that while the . By the memoryless property of the exponential distribution, we have that . Thus, we want to find . Thus, the probability of getting 20 more miles is 37%.
2. We now have that and wish to calculate the conditional probability .

**Question #37:** If X is uniformly distributed over , find (a) and (b) the density function of the random variable .

1. We have that and wish to calculate .
2. We begin by finding the CDF of , where we have for all . We then differentiate the CDF to obtain the PDF as for all . Therefore, we have found that .

**Question #40:** If X is uniformly distributed over , find the density function of .

* First we start with the cumulative density function , since we want to find the PDF for . We have , which is the CDF with respect to . After taking the derivative to get the PDF we obtain by the chain rule. But since , we know that and the domain changes to , so we have that the PDF of is .