**Chapter #4 – Discrete Random Variables**

**Question #1:** Two balls are chosen randomly from an urn with 8 white, 4 black and 2 orange balls. Suppose that we win $2 for each black ball selected and we lose $1 for each white ball selected (nothing happens with the orange balls). Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value?

* There are 14 total balls in the urn. The possible values are , which are computed from the six unique combinations of balls that can be drawn. For example, two white balls will yield –$2 while one black and one white ball will yield winnings of $1. The following are the desired probabilities.
  + (white + white)
  + (orange + white)
  + (orange + orange)
  + (white + black)
  + (orange + black)
  + (black + black)

**Question #5:** Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X?

* Let denotes the number of heads and denotes the number of tails when a coin is tossed times. Since , we have or . Therefore, we can see that for . In other words, the possible values for the random variable are .

**Question #6:** In Problem 5, for if the coin is assumed fair, what are the probabilities associated with the values that X can take on?

* We see that . After listing the outcomes, the following are the desired probabilities for the four values of the random variables.

**Question #13:** A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .3, and his second will lead independently to a sale with probability .6. Any sale made is equally likely to be either for the deluxe model, which costs $1000, or the standard model, which costs $500. Determine the probability mass function of X, the total dollar value of all sales.

* If we let denote the event of no sale, denote a standard sale and denote a deluxe sale, then the sample space while the random variable assigns the following real numbers to these outcomes:
* If we recall that , and , the probabilities to include in the probability mass function are calculated as follows.

**Question #21:** Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let denote the number of students on her bus. (a) Which of or do you think is larger? Why? (b) Compute and in this experiment.

1. In the first sampling method, we draw one of 148 students at random. Naturally, the probability of choosing a student from the fullest bus is higher than the probability of drawing a student from the other buses. Thus, the fuller buses are weighted higher than the other buses, so we would expect to be larger. In the second method, one of the four bus drivers is randomly selected. Each bus driver is equally likely to be chosen! Each bus is weighted the same in this came, therefore, . Equality holds when each bus contains the same number of students.

**Question #22:** Suppose that two teams play a series of games that ends when one of them has won games. Suppose that each game played is, independently, won by team A with probability . Find the expected number of games that are played when we have (a) and (b) . Also, show in both cases that this number is maximized when .

1. Let denote the number of games played. When , the random variable can be either 2 (one teams wins both) or 3 (the teams split the first two games and winner of the third wins the game). Therefore, we can compute the expected value of as follows: . To find where the expected value is maximized, we differentiate and set equal to zero to solve for the probability : (this is a maximum since the second derivative is negative, .
2. When , the random variable can be either 3, 4 or 5. We thus calculate as follows . Then we see that .

**Question #36:** Consider Problem 22 above with . Find the variance of the number of games played, and show that this number is maximized when .

* . We then calculate the derivative to optimize: .

**Question #26:** One of the numbers 1 through 10 is randomly chosen. You are to try to guess the number chosen by asking questions with “yes–no” answers. Compute the expected number of questions you will need to ask in each of the following two cases: (a) Your ith question is to be “is it i?” for . (b) With each question you try to eliminate one half of the remaining numbers, as nearly as possible.

1. Let be the number of questions asked. It then follows that for all . This, we have that .
2. Consider the following strategy: divide what you have in halves and ask, “is it greater than or equal to ?” where is the middle of the numbers. For instance, your first question is, “is it greater than or equal to 5.5?”. (All other strategies of this type are similar. But you need to be consistent.) Let denote the random number, so that for all . If , then and if , then we see that . Therefore, we have the following results:
   * .

**Question #41:** A man claims to have extrasensory perception. As a test, a fair coin is flipped 10 times and the man is asked to predict the outcome in advance. He gets 7 out of 10 correct. What is the probability that he would have done at least this well if he had no ESP?

* If we let be the number of correct guesses, then we see . The probability that he would have done at least as well as seven correct guess is thus given by .

**Question #46:** Suppose that it takes at least 9 votes from a 12-member jury to convict a defendant. Suppose also that the probability that a juror votes a guilty person innocent is 0.2, whereas the probability that the juror votes an innocent person guilty is 0.1. If each juror acts independently and if 65% of the defendants are guilty, find the probability that the jury renders a correct decision. What percentage of defendants is convicted?

* Let the events be (vote innocent), (vote guilty), (guilty) and (innocent), so we are given , , and . Thus, if is conviction, what we want to calculate is , where
* This, the answer is .

**Question #56:** How many people are needed so that the probability that at least one of them has the same birthday as you is greater than ?

* The number of people in a random collection of size that have the same birthday as yourself is approximately Poisson distributed with mean . Hence, the probability that at least one person has the same birthday as you is approximately equal to . Since when we have , we have when so .

**Question #71:** Consider a roulette wheel consisting of 38 numbers 1 through 36, 0 and double 0. If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that (a) Smith will lose his first 5 bets, (b) his first win will occur on his fourth bet?

**Question #82:** A purchaser of transistors buys them in lots of 20. It is his policy to randomly inspect 4 components from a lot and to accept the lot only if all 4 are nondefective. If each component in a lot is, independently, defective with probability 0.1, what proportion of lots is rejected by the purchaser?

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