

Homework 4

Math 5010-30, Fall 2009

1. Determine a reasonable sample space for a public opinion poll (with sample size n) like the ones discussed in class. How many elements are in this sample space?
2. How would the statistical analysis of an election change if there were more than two viable candidates?
3. *Time* magazine (July 15, 2002) quoted a poll of 10,000 Americans in which only 4% said they were vegetarians.
 - (a) What has to be assumed about this sample in order to construct a confidence interval for the population proportion of vegetarians?
 - (b) Construct a 99% confidence interval for the population proportion. Explain why the interval is so narrow, even the confidence level is high.
 - (c) In interpreting this confidence interval, can you conclude that fewer than 10% of Americans are vegetarians? Explain your reasoning.
4. As part of the 2000 census, the Census Bureau surveyed 700,000 households to study transportation to work. They reported that 76.3% drove alone to work, 11.2% carpoled, 5.1% took mass transit, 3.2% worked at home, 0.4% bicycled, and 3.8% took other means.
 - (a) With such a large survey, explain why the margin of error for any of these values is extremely small.
 - (b) The survey also reported that the mean travel time to work was 24.3 minutes, compared to 22.4 minutes in 1990. Explain why this is not sufficient information to construct a confidence interval for the population mean (in 1990). What else would you need?
5. Let $X \sim \text{POI}(\lambda)$ (i.e., X is a Poisson random variable with parameter λ).
 - (a) Compute $E[X]$.
 - (b) Compute $\text{Var}(X)$.
6. Show that the standard deviation of the sample proportion is always less than or equal to $\frac{1}{2\sqrt{n}}$ for all values of p . That is:

$$SD(\hat{p}) \leq \frac{1}{2\sqrt{n}} \text{ for all } p \in [0, 1]. \quad (1)$$

Explain why this implies that the margin of error formula

$$\text{MOE} = \frac{1}{\sqrt{n}} \quad (2)$$

always yields conservative confidence intervals.

7. Suppose that you are in charge of public opinion polling for candidate A in the upcoming congressional elections. Having conducted several polls already, you have reason to believe that approximately 60% of the population supports candidate A. Candidate A, however, has recently been involved in a minor scandal and would like to know if his support is wavering. What size sample do you need to use to be able to give your boss a 95% confidence interval with a margin of error of less than 2%?
8. Suppose that rather than conducting an opinion poll, in which each sample subject is asked to give a binary response (yes = 1 or no = 0), you conduct a survey in which each respondent is asked for a numerical response. For example, the score a student received on a test or a person's height or weight.

Note that each of these responses can be thought of as an independent realization of a random variable with mean μ and variance σ^2 .

Construct a reasonable estimator of this population mean μ . Is this estimator random or nonrandom? Can you compute the expectation and/or variance of your estimator?

Hint: To estimate the population proportion we computed \hat{p} , the proportion of the sample who gave a specified response to our poll question. Can you do something similar with the sample mean?