Dan Ciubotaru: **Spherical unitary representations for split groups.**

The aim of this talk is to present some recent progress in the problem of identifying the spherical unitary representations of groups defined over local fields.

The setting is the following: let G be the $\mathbb{F}$-points of a reductive algebraic group defined over the real or p-adic field $\mathbb{F}$, and let K be a maximal compact subgroup, $B = AN$ be a Borel subgroup with $G = KB$. An admissible representation (π, V) is called *spherical* if $V^K \neq 0$. Let us assume that G is split. The spherical irreducible representations appear as the (unique) spherical subquotients $L(\chi)$ of principal series $X(\chi)$ induced from *unramified* characters χ of the maximal (split) torus A ($\chi|_{A\cap K} = 1$). Moreover, results of Knapp in the real case, and Barbasch-Moy in the p-adic case, allow one to consider only *real* parameters χ, thus identifying the parameter sets in the two cases.

The question of unitarity for a (Hermitian) spherical parameter χ can be formulated in terms of the signature of intertwining operators $a_\mu(\chi) : (V_\mu^{A\cap K})^* \to (V_\mu^{A\cap K})^*$, for K-representations (μ, V_μ) in the real case, and $a_\tau(\chi) : (V_\tau)^* \to (V_\tau)^*$, for W-representations (τ, V_τ) in the p-adic case. The space $(V_\mu^{A\cap K})^*$ carries a natural representation of the Weyl group, call it $\tau(\mu)$. The idea of *petite K-types* (Barbasch-Vogan) is that for certain K-representations μ (the definition will be made precise), $a_\mu(\chi) = a_{\tau(\mu)}(\chi)$.

In the p-adic setting, D. Barbasch for classical groups, and D. Barbasch and the presenter for exceptional groups determined the spherical unitary dual, and showed that the W-representations coming from petite K-types (i.e., of the form $\tau(\mu)$) are sufficient for ruling out all non-unitary spherical representations. Therefore, one obtains a set of parameters for the real case, which conjecturally should be unitary. We should mention that this is actually a theorem for classical groups (Barbasch), but not yet determined for all the exceptional groups.

We will present in detail the constructions mentioned above, and explain the main features of the classification of spherical unitary representations.