Remarks on the Geometry of the Springer fiber and Applications. The $SU(p,q)$-case.

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We assume that $G = SL(p+q, \mathbf{C})$ with real form $G_o = SU(p,q)$ and fix a Cartan involution θ. The variety $\mathcal{B}$ of borel subalgebras in $\mathfrak{sl}(p+q, \mathbf{C})$ is acted upon by $G^\theta = S(GL(p, \mathbf{C}) \times GL(q, \mathbf{C}))$ with finitely many orbits. We denote such orbits by $\{Z_i\}$. Let $T^*(\mathcal{B})$ denote the cotangent bundle to $\mathcal{B}$ and let $\mu : T^*(\mathcal{B}) \to \mathcal{N}^*$ denote the moment map. The fiber, $\mu^{-1}(\xi)$, at a point $\xi \in \mathcal{N}^*$ is known as the Springer fiber.

For each Z_i, we write $T^*_{Z_i}(\mathcal{B}) \subset T^*(\mathcal{B})$ for the conormal bundle to Z_i. It is known that $\mu(\overline{T^*_{Z_i}(\mathcal{B})})$ is the closure of a nilpotent K-orbit, $\mathcal{O}$. This associates a nilpotent K-orbits to a K-orbits in the flag variety. When a K-orbit Z is associated to a nilpotent orbit $\mathcal{O}$, the intersection $\mu^{-1}(\xi) \cap T^*_Z(\mathcal{B})$ is dense in a unique irreducible component of the Springer fiber.

In this talk we discuss $\mu^{-1}(\xi) \cap T^*_Z(\mathcal{B})$ when Z is a closed orbit in $\mathcal{B}$. We give an explicit description of such intersections in terms of the structure of the groups involved. An algorithm will be described that computes the multiplicity of $\overline{\mathcal{O}}$ in the Associated Cycle of representations in the discrete series of $SU(p,q)$.