Six Degrees of Kevin Bacon:

Is it really a small world after all?

Science Day
November 13, 2010

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University of Utah

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High School Summer Program

What does “It’s a small world” mean?

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This is interesting if there are a large number of people and a seemingly small number of connections.

The real question

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Is it an enormous world that is so highly stratified that it gives the false impression of being small?

Or is an enormous world that is genuinely small after all?

A famous experiment

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The catch is that you can only send the package to someone you know on a first-name basis (and the same restrictions applies to the recipient).

Miligram’s findings

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This gave rise to the now universal belief that any two strangers can be connected by at most six degrees of separation.

It also answers our original question: the world appears to actually be small.

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There are other (difficult to quantify) phenomena at work here: identity and searchability, for instance.

A newer experiment

A Columbia University research team (Dodds, Muhamad, Watts) are conducting a much more careful version of Milgram's experiment using email.

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A Columbia University research team (Dodds, Muhamad, Watts) are conducting a much more careful version of Milgram's experiment using email.

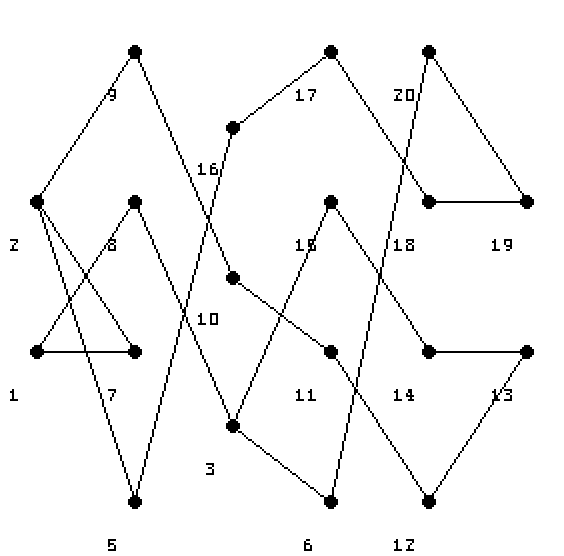
You can participate! (Google "small world experiment".)

Some partial results have been published.

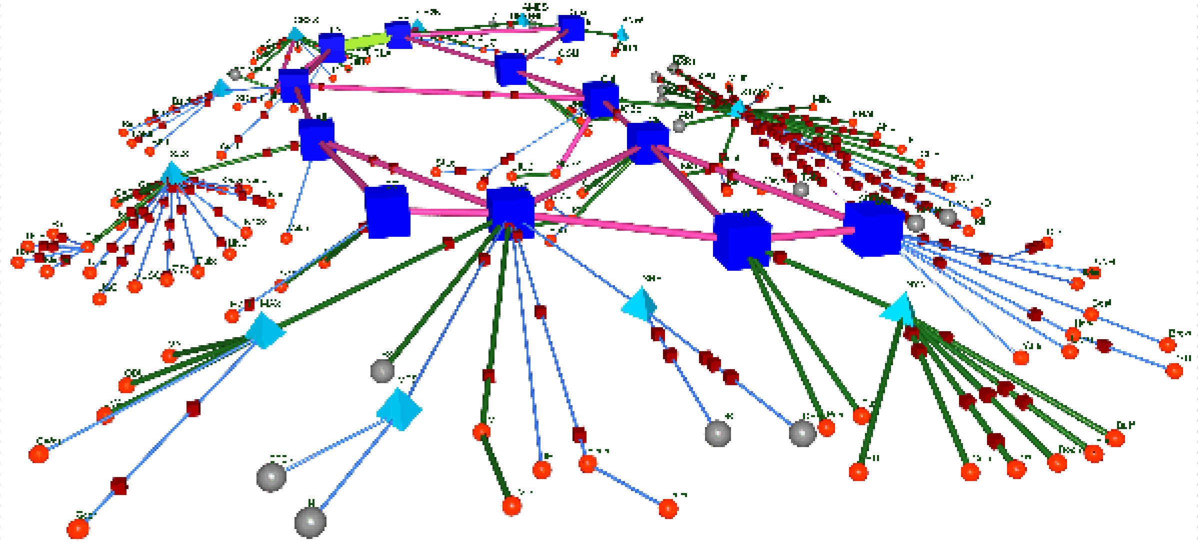
A more general setting: graphs

A graph consists of a vertex set, together with a set of edges connecting some vertices.

Examples of graphs



Examples of graphs



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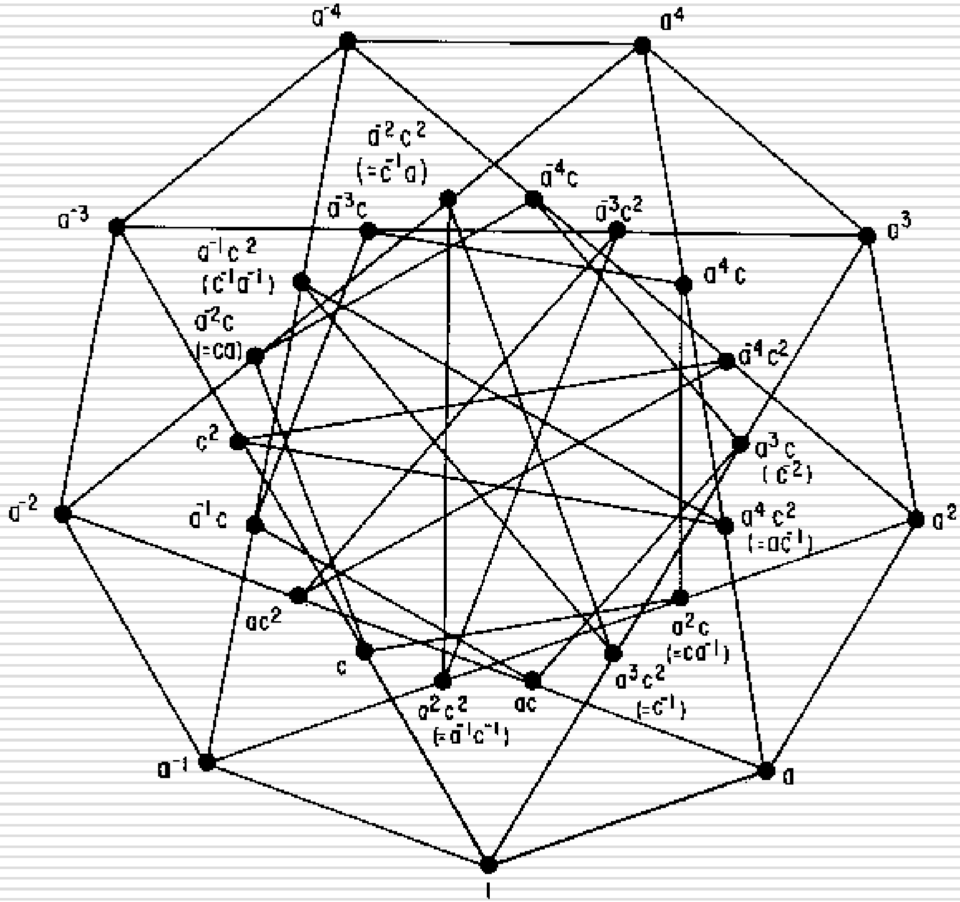

Examples of graphs

The graph of the world: vertices are people, and an edge connects two people if they know each other.

Examples of graphs

The Kevin Bacon Graph:

Vertices are actors.

Two vertices are connected if they appeared in a movie together.

Distance in graphs

The *distance* between two vertices is the minimum number of edges needed to connected the vertices.

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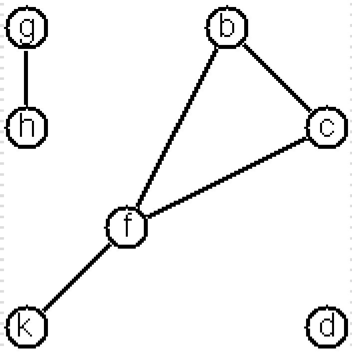

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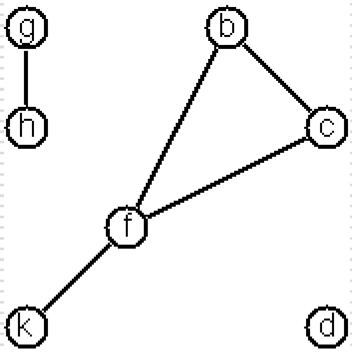

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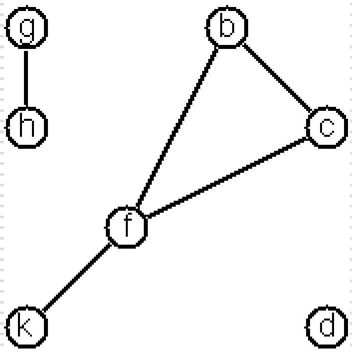

d(g,h) = 1

d(k,b) = 2

d(g,b) = infinite

Bacon Numbers

Define the Bacon number of an actor to be the distance to Kevin Bacon in the Kevin Bacon Graph.

The oracle of Kevin Bacon

Britney Spears has a Bacon number of 2.

Britney Spears was in Austin Powers in Goldmember (2002) with Greg Grunberg

Greg Grunberg was in Hollow Man (2000) with Kevin Bacon

The oracle of Kevin Bacon

Leonardo DiCaprio has a Bacon number of 2.

Leonardo DiCaprio was in Celebrity (1998)
with Gerry Becker

Gerry Becker was in Trapped (2002)
with Kevin Bacon

The oracle of Kevin Bacon

Charlie Chaplin has a Bacon number of 3.

Charlie Chaplin was in Countess from Hong Kong (1967) with Marlon Brando

Marlon Brando was in Hearts of Darkness: A Filmmaker's Apocalypse (1991) with Colleen Camp

Colleen Camp was in Trapped (2002) with Kevin Bacon

Bacon Numbers

Bacon number	Number of actors
0	1
1	1703
2	135839
3	370657
4	90057
5	7154
6	922
7	94
8	2

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Not really. The KBG is pretty random and there are lots of short cuts.

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For a random actor x, the average x-number is about 3.3. (KB's was 2.9.)

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Could play this game with almost *anyone:*
For a random actor x, the average x-number is about 3.3. (KB's was 2.9.)

The center of the universe is Rod Steiger (average Rod Steiger number is 2.5.)

Characteristic path length

Given a *connected* graph with n vertices, define its *characteristic path length* to be the average distance between pairs of vertices.

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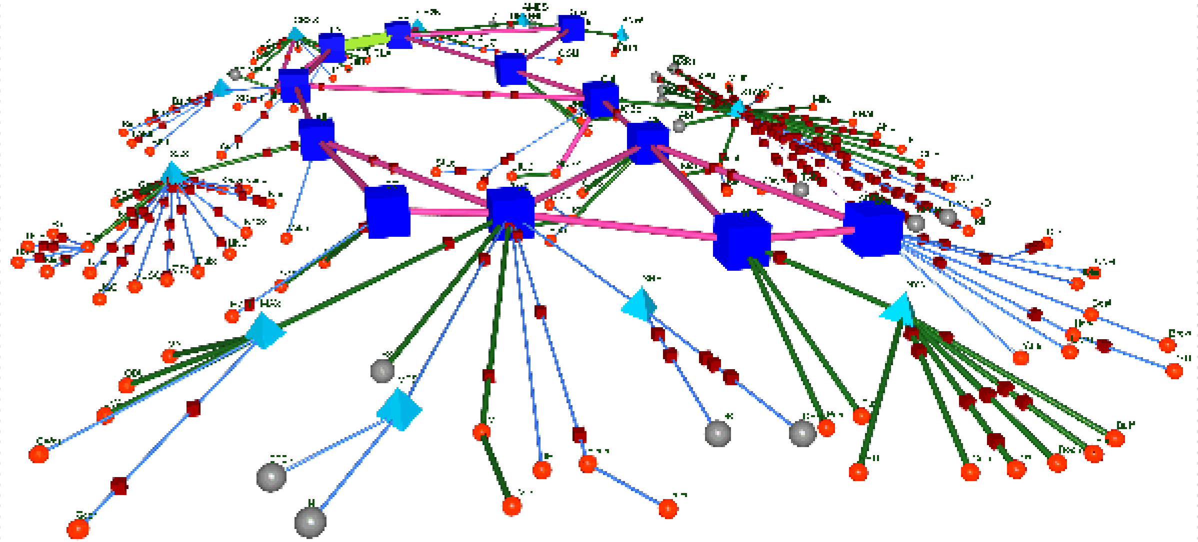
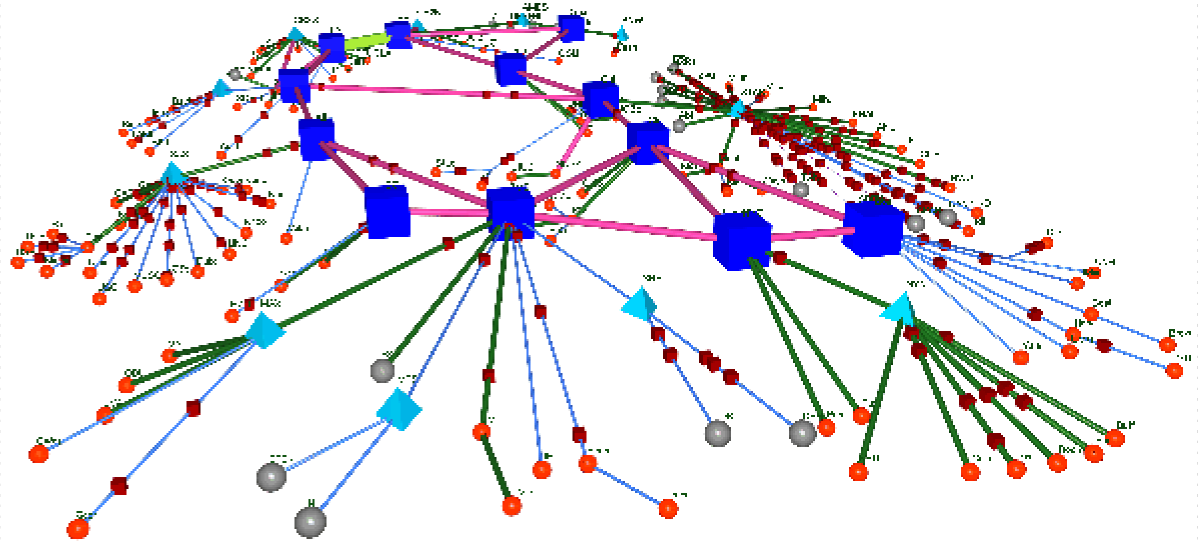
$$L = \frac{1}{n^2} \sum_i \sum_j d(i, j)$$

Example: Characteristic path length



Example: Characteristic path length

Large L



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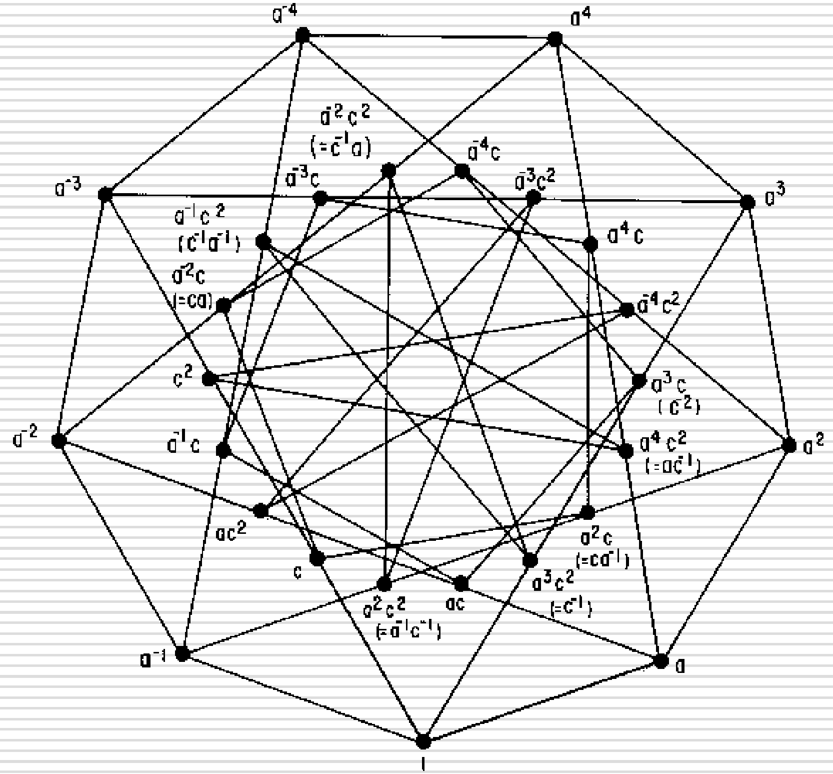
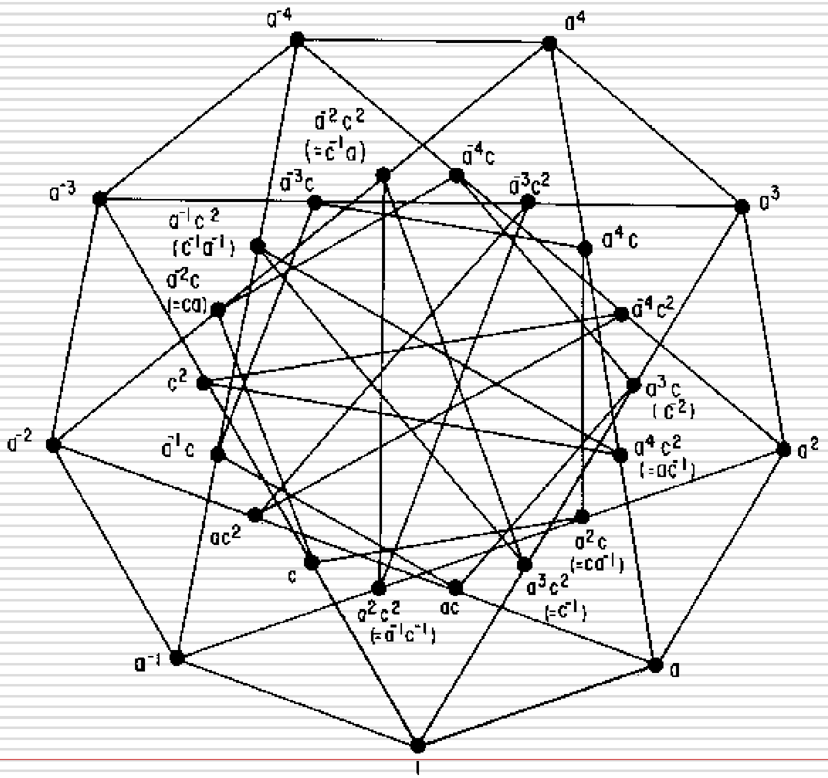


Example: Characteristic path length

Small L



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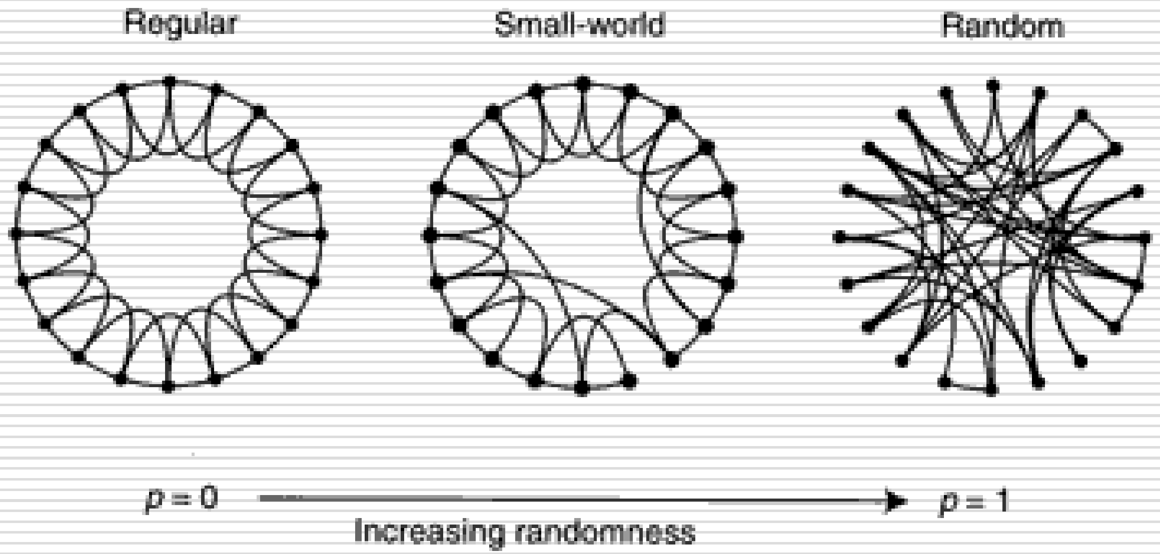

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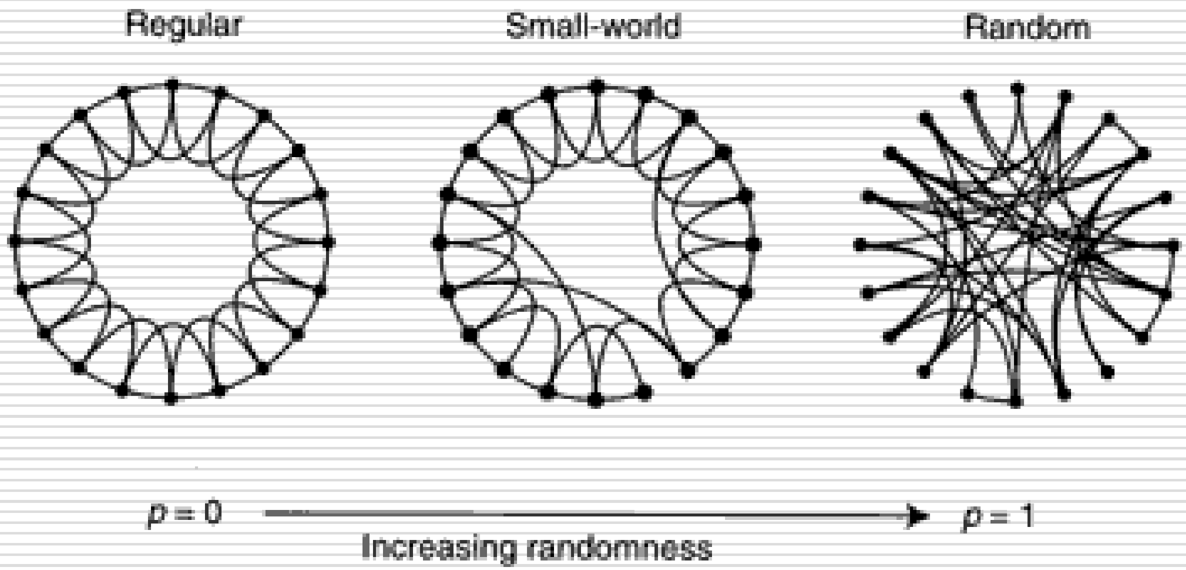

L large

L small

Roughly…

Random graphs have small L's.

Highly clustered graphs (e.g. Caveman Type graphs) have small L's.

The clustering coefficient C

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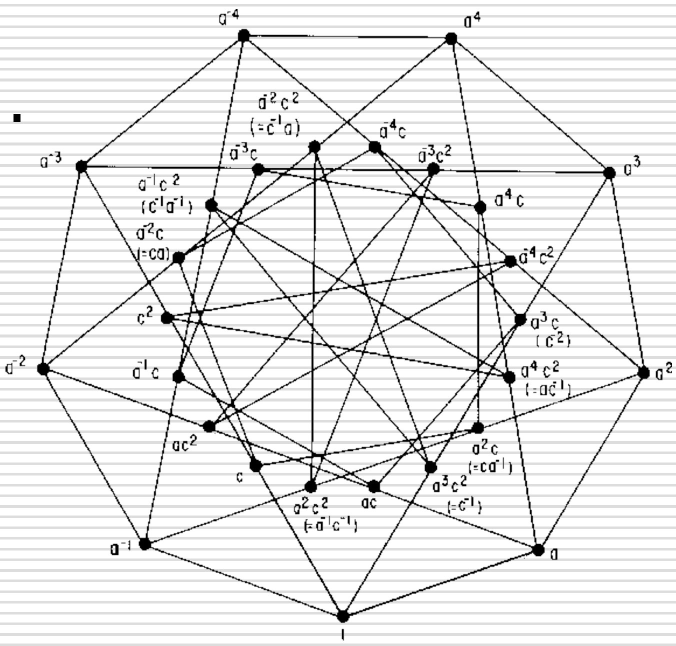
Define the clustering coefficient at a vertex V to be the fraction of the pairs of vertices connected to V which are themselves connected to each other.

i.e.: Do your friends know each other?

The clustering coefficient C

Define the clustering coefficient of a graph to be the average (over all vertices) of the the vertex clustering coefficients.

Clustering coefficient examples



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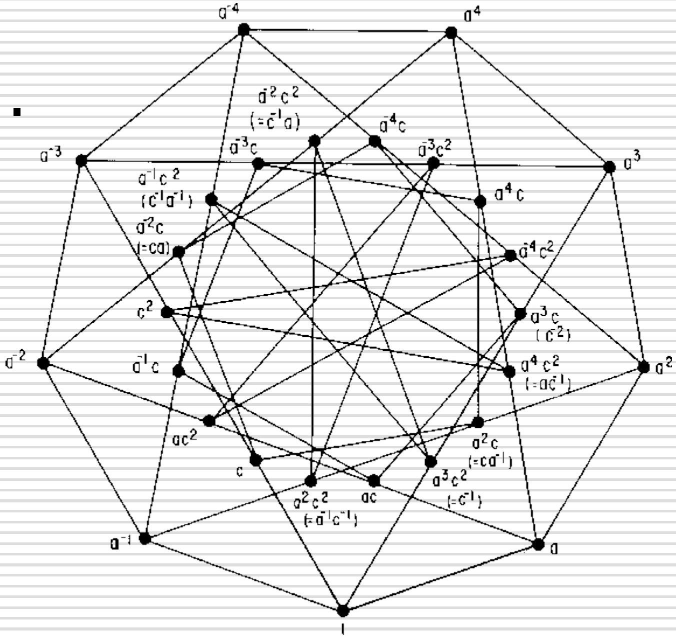
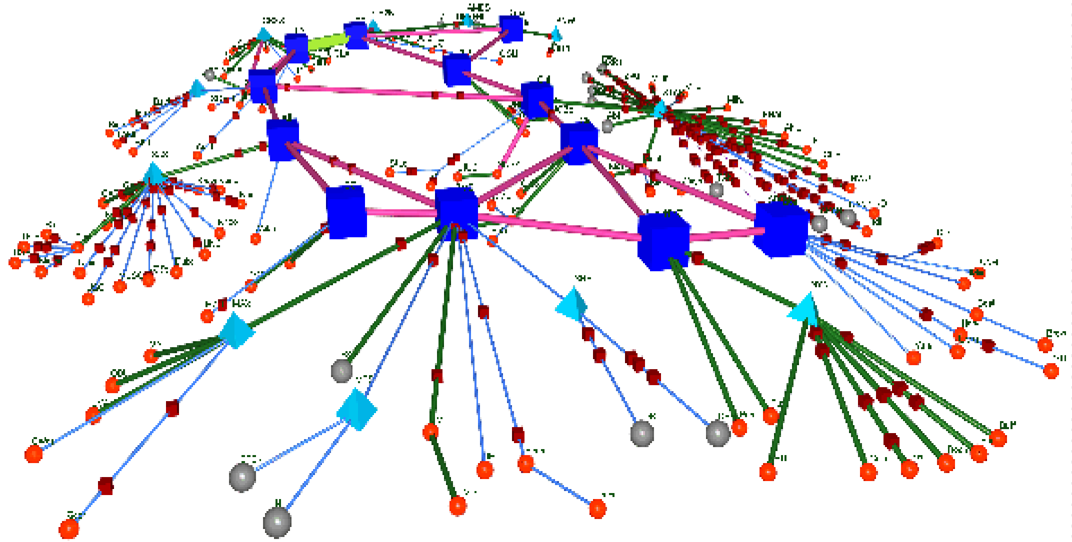


C = 1

Clustering coefficient examples



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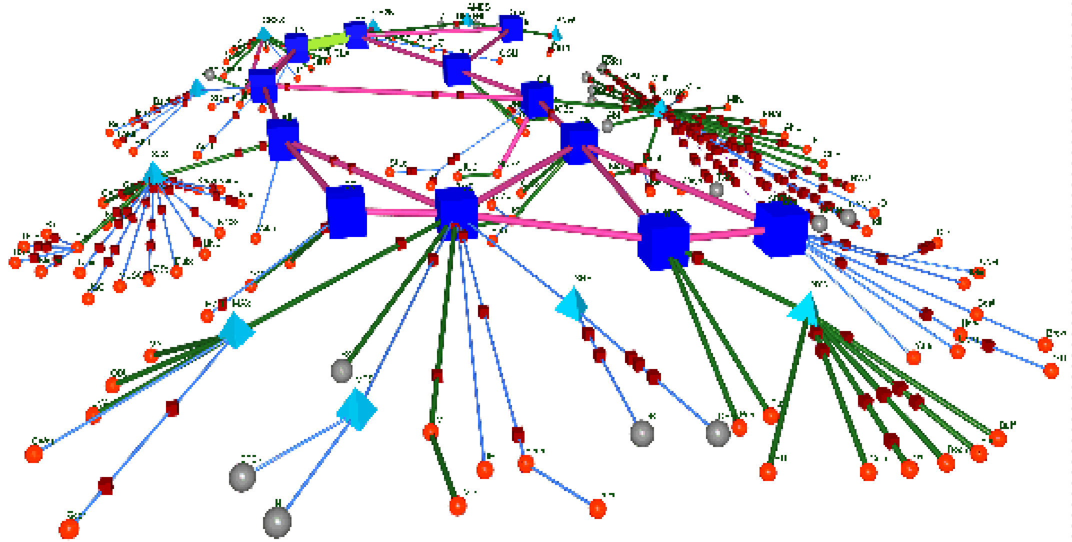

C small

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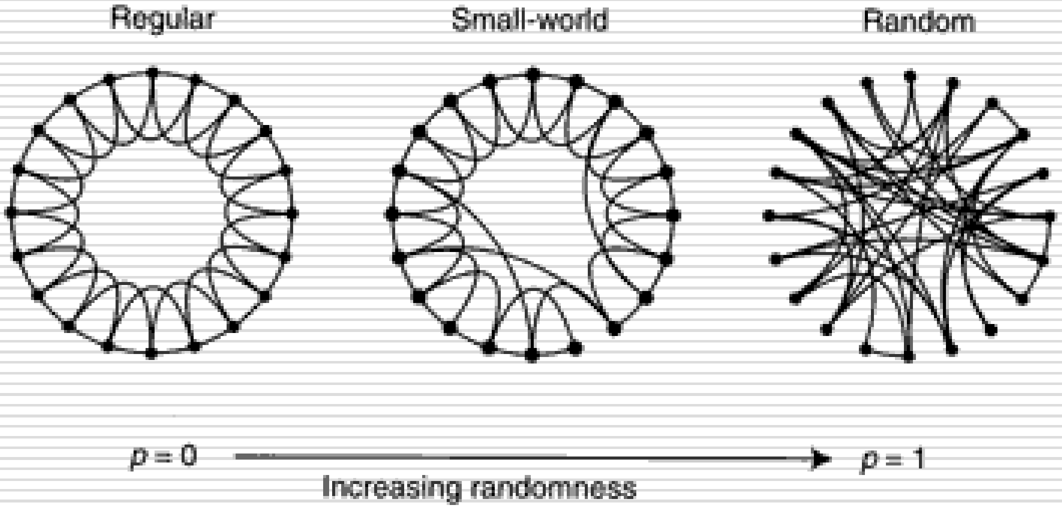

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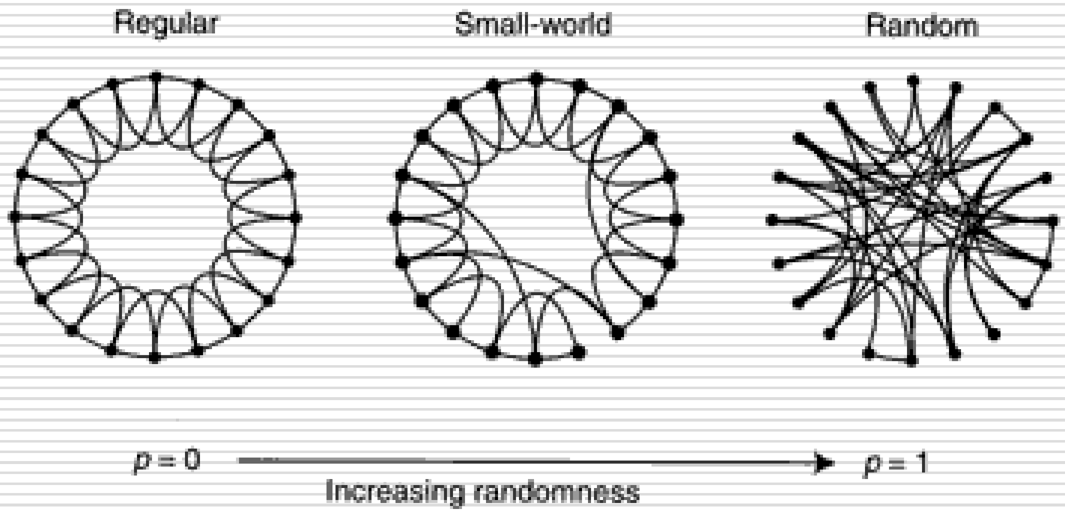

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These are “small worlds that aren’t very stratified.”

Caveman-type graphs have large C’s (and large L’s).

These are “highly stratified worlds that aren’t small.”

The question remains

Can we find a graph that is highly stratified (C close to 1) yet which exhibits the small world property (L small).

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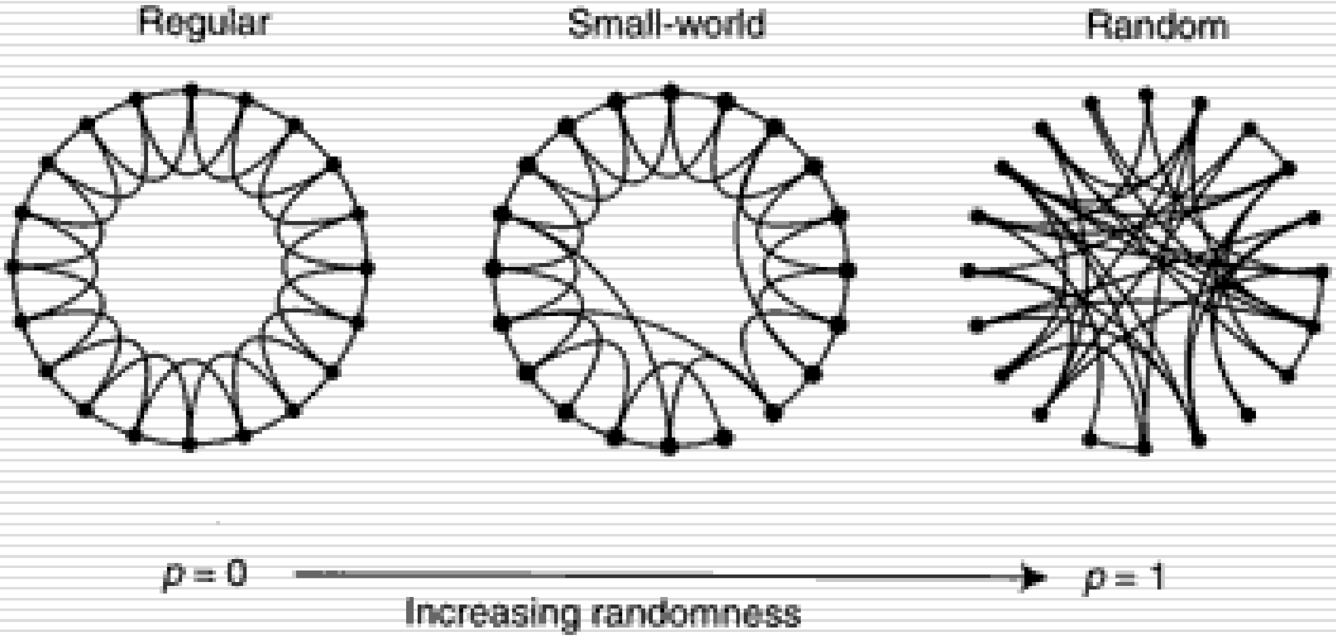
If our world is really small, it gives that kind of graph.

Highly Stratified Small Worlds



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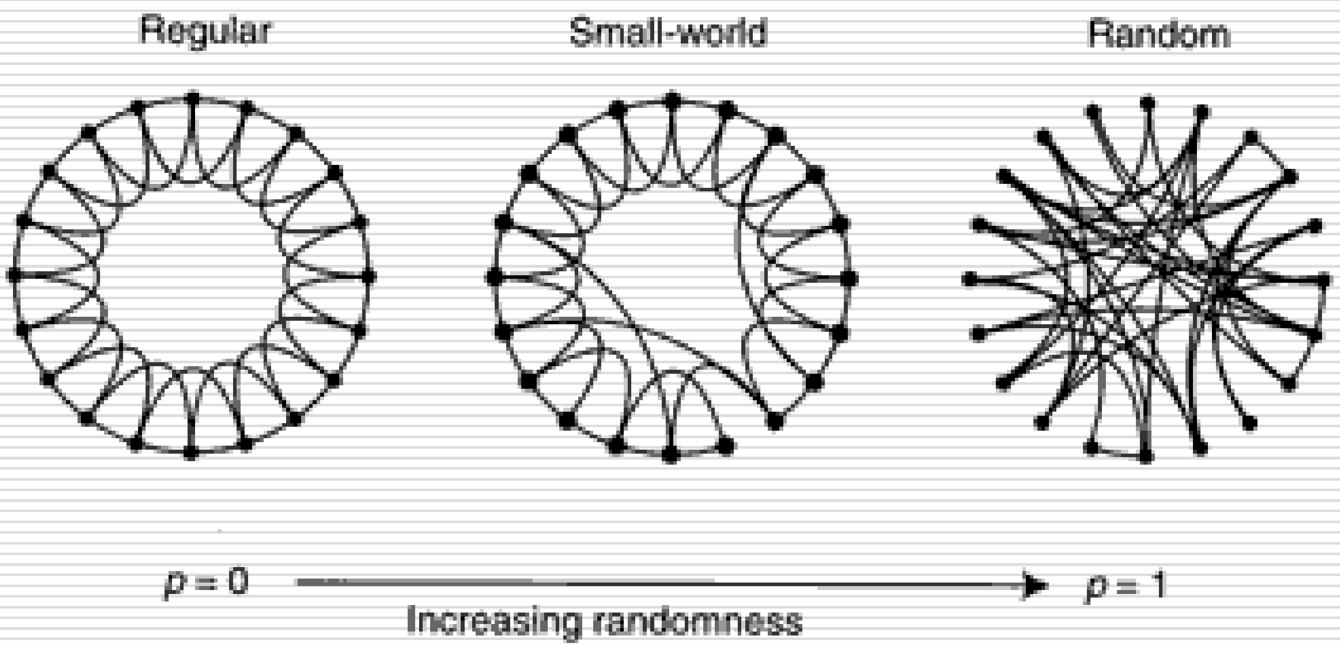

Small L
Large C

Other questions…

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Preview of coming attractions: The "Google weight" of a vertex. (See Jim Carlson's talk next week).