WEYL GROUP REPRESENTATIONS AND NILPOTENT ORBITS

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1. INTRODUCTION

In [B-V2] and [B-V3] two related problems are studied for complex semisimple groups. One, is to classify the primitive ideals in the enveloping algebra. The other is to study Fourier inversion of unipotent orbital integrals.

This is carried out explicitly for regular integral infinitesimal character. The connection between the two problems is established via characters of irreducible admissible $(\mathfrak{g}, K)$ modules and their wave front set or asymptotic support.

In these notes we would like to illustrate how the methods involved in [B-V2, 3] generalize to real rank groups in order to compute the cells (definition 2.5) and Fourier inversion for unipotent orbits. There are many technical complications and the results are far from complete. In order to keep the notation to a minimum and to be as clear as possible we will mainly restrict attention to $G = U(p,q)$ where the most complete results are known. (To give an idea how much more difficult the problem is for general groups, it is useful to compare the solution of the problem of classifying primitive ideals for type A_n, due to Joseph, to the solution of the same problem for an arbitrary semisimple Lie algebra).

The paper is organized as follows. In section 1 we introduce notation. In section 2 we parametrize the representations with a fixed regular integral infinitesimal character. We then study to coherent continuation representation and introduce the notion of a cell. In section 3 we review some results from [B-V1] on asymptotic support in order to calculate it for a special class of representations. In section 4 we apply the calculations in sections 1-3 to the case of $U(p,q)$.

We consider G a connected reductive real group such that $G \subseteq G_c$, a complexification of G. Let $\mathfrak{g}_0$ be the Lie algebra of G and θ a Cartan involution. Let $\mathfrak{g}_0 = \mathfrak{k}_0 + \mathfrak{p}_0$ be the Cartan decomposition and $\mathfrak{g}, \mathfrak{k}, \mathfrak{p}$ the respective complexifications.

For $H \subseteq G$, a θ-stable Cartan subgroup, we write $\mathfrak{h}_0$ for its Lie algebra, $\mathfrak{h}$ for the complexification. Let $\Delta(\mathfrak{g}, \mathfrak{h})$ be the root system. We also write

$$H = TA = (H \cap K)(H \cap \exp \mathfrak{p}_0)$$

for the Cartan decomposition.

We make the following assumptions on G.

a) all Cartan subgroups are connected

b) rank $\mathfrak{g}$ = rank $\mathfrak{k}$.

The compact torus will be denoted by T. We will also use the notation $\mathfrak{h}_a$, $\Delta(\mathfrak{g}, \mathfrak{h}_a)$ etc. to denote the "abstract" Cartan subalgebra, root system and so on.

We will denote by W, the complex Weyl group of H, by $W^\theta(H) \subseteq W$, the θ-stable part of the Weyl group and by $W(H) \subseteq W^\theta(H)$ the real Weyl group of H.

2. THE COHERENT CONTINUATION REPRESENTATION

We need to parametrize all representations with fixed regular integral infinitesimal character. Without loss of generality we can assume that this infinitesimal character is ρ, in other words the same as the infinitesimal character of the trivial representation.

PROPOSITION 2.1 The set of equivalence classes of irreducible admissible representations with trivial infinitesimal character is in one-to-one correspondence with the set of K-conjugacy classes of positive systems $\Delta^+ \subseteq \Delta(\mathfrak{g}, \mathfrak{h})$ where $\mathfrak{h}$ is a θ-stable Cartan subalgebra.

PROOF. This is the Langlands classification applied to the case of a group G satisfying conditions a) and b) together with proposition 2.7 in [V2].

Given a regular integral $\lambda \in \mathfrak{h}'$, we will denote by $\pi(\lambda)$ the induced representation from a discrete series and call it the standard representation. We will denote by $\overline{\pi}(\lambda)$ the corresponding Langlands quotient. We let F be the group of virtual characters with trivial infinitesimal character. Then W_a acts on F in the following way.

Let $\Lambda \subseteq \mathfrak{h}_a^*$ be the integral lattice.

Definition 2.2. A coherent family of virtual characters $\{\Theta(\lambda)\}_{\lambda \in \Lambda}$ is a set of virtual characters $\Theta(\lambda)$ with infinitesimal character λ, satisfying

(2.2.1) if λ is dominant for a fixed positive system Δ_a^+, then $\Theta(\lambda)$ is 0 or the character of an irreducible representation; and the latter is the case if λ is nonsingular

(2.2.2) if F is finite dimensional,

$$\Theta(\lambda) \cdot \Theta(F) = \sum_{\mu \in \Delta(F)} \Theta(\lambda + \mu) .$$

Given any character of an irreducible representation there is a unique coherent family it belongs to.

Definition 2.3. For $w \in W_a$, we define $w\,\Theta(\lambda) = \Theta(w\lambda)$.

This action of W_a on F is called the coherent continuation representation.

We now decompose this representation. For any Θ-stable Cartan subgroup H we define a 1-dimensional representation of $W(H)$ in the following way. Let $\Delta^{i\mathbb{R}} \subseteq \Delta(\mathfrak{g},\mathfrak{h})$ be the subsystem of imaginary roots. Let $\mathfrak{h}_I = \text{lin span } \{H_\alpha \in \mathfrak{h}\colon \alpha \in \Delta^{i\mathbb{R}}\}$. Then $W(H) = \det \mathrm{Ad}w|_{\mathfrak{h}_I}$ for any $w \in W(H)$.

PROPOSITION 2.4

$$F \simeq \bigoplus_{\substack{H \text{ conjugacy} \\ \text{class of } \theta\text{-stable} \\ \text{Cartan subgroup}}} \operatorname{Ind}_{W(H)}^{W} \det{}_I$$

PROOF. We first describe $W(H)$ ([V3]proposition). Let $\Delta^{\mathbb{R}} \subseteq \Delta(\mathfrak{g},\mathfrak{h})$ be the system of real roots and fix positive systems

$\Delta^{\mathbb{R},+} \subseteq \Delta^{+}(\mathfrak{g},\mathfrak{h})$, $\Delta^{i\mathbb{R},+} \subseteq \Delta^{+}(\mathfrak{g},\mathfrak{h})$. Let

$$\rho^{i\mathbb{R}} = \tfrac{1}{2} \sum_{\alpha \in \Delta^{\mathbb{R},+}} \alpha$$

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Then define

$$\Delta^{q} = \{\alpha \in \Delta(\mathfrak{g},\mathfrak{h}) : (\alpha,\rho^{i\mathbb{R}}) = 0\}$$

$$\Delta^{f} = \{\alpha \in \Delta : (\alpha,\rho^{\mathbb{R}}) = 0\}$$

$\Delta^{\mathbb{C}} = \Delta^{q} \cap \Delta^{f}$ and $W^{\mathbb{R}}$, $W^{\mathbb{C}}$, $W^{i\mathbb{R}}$ accordingly. Then

$$W(H) \simeq (W^{\mathbb{C}})^{\theta^{W(H)}} \ltimes (W^{\mathbb{R}} \times W(G^{A},H))$$

where G^{A} is the centralizer of A in G and $W(G^{A},H) \subseteq W^{i\mathbb{R}}$.

A natural basis for F is given by the characters of the standard representations $\pi(\gamma)$. We filter F according to dim A ,

$$F = F_{0} \supseteq F_{1} \supseteq \cdots \supseteq F_{i} \supseteq \cdots$$

where F_{i} is spanned by the $\pi(\gamma)$ such that γ is associated to a Cartan subgroup with split component of dimension greater than or equal to i . This is compatible with the action of W_{a} .

The discrete series character identities in [S] and the invariance of induced representations by $W^{\mathbb{R}}$ show that the coherent continuation representation on F_{i}/F_{i+1} coincides with $\det_{I}$ on $W^{\mathbb{R}} \times W(G^{A},H)$. Remains to check the action of $(W^{\mathbb{C}})^{\theta}$. Suppose $\gamma = (\delta,\nu)$ where $\delta \in \mathfrak{t}^{*}$ corresponds to a discrete series parameter on G^{A} , $\nu \in \mathfrak{a}^{*}$. Then if $w \in (W^{\mathbb{C}})^{\theta}$, $w(\delta,\nu) = (\delta^{1},\nu^{1})$ in such a way that $\pi(\delta,\nu)$ is equivalent to $\pi(\delta^{1},\nu^{1})$. On ther other hand it can be seen that $\det_{I} w = 1$. Since also $\dim F_{i}/F_{i+1} = |W/W(H)|$, the result follows from the fact that any representaion of a finite group on a filtered vector space is equivalent to the representation on the associated graded vector space.

Definition 2.5. <u>We say</u> $\overline{\pi}(\gamma_1)$ <u>and</u> $\overline{\pi}(\gamma_2)$ <u>belong to the same cell and write</u> $\overline{\pi}(\gamma_1) \sim \overline{\pi}(\gamma_2)$ <u>if there are finite dimensional representations</u> F_1 , F_2 <u>such that</u> $\overline{\pi}(\gamma_1)$ <u>is a composition factor of</u> $\overline{\pi}(\gamma_2) \otimes F_2$ <u>and</u> $\overline{\pi}(\gamma_2)$ <u>is a composition factor of</u> $\overline{\pi}(\gamma_1) \otimes F_1$. <u>We denote by</u> $C(\gamma)$ <u>the cell of</u> γ <u>and similar to [B-V2,3]</u>,

$$K(\gamma) = \{\gamma^{1} : \text{there is } F^{1} \text{ finite dimensional such that } \gamma^{1} \text{ occurs in } \overline{\pi}(\gamma) \otimes F^{1}\}$$

$$K^{+}(\gamma) = \{\gamma^{1} : \gamma^{1} \in K(\gamma),\ \gamma^{1} \notin C(\gamma)\} .$$

By abuse of notation we also denote the corresponding vector spaces generated by the characters by $K(\gamma)$ and $K^{+}(\gamma)$. Let

$$V(\gamma) = K(\gamma)/K^{+}(\gamma)$$

<u>PROPOSITION 2.6.</u> $V(\gamma)$ is stable under the action of W and has a basis formed of the characters $\overline{\pi}(\gamma)$ of $\gamma^{1} \in C(\gamma)$.

<u>PROOF</u>. We refer to [BV-3], 2.8-2.12 for details.

3. NILPOTENT ORBITS AND REPRESENTATIONS

In this section, we review the notion of asymptotic support as defined in [B-V1]. As is apparent in [B-V2,3] in order to determine the cells one needs to know the induced nilpotent orbits. We define the appropriate notion and the representation that should give them.

Any character Θ_{π} can be lifted to an eigendistribution θ_{π} in a neighborhood of 0 in $\mathfrak{g}$. For $f \in C_{c}^{\infty}(\mathfrak{g})$ and $t > 0$ we let $f_{t}(x) = t^{-\dim \mathfrak{g}} f(t^{-1}x)$. Then $\theta_{\pi}(f_{t})$ has an asymptotic expansion (like a Taylor series)

$$\theta_{\pi}(f_{t}) = \sum_{i=-r}^{\infty} t^{i} D_{i}(f) \quad \text{as} \quad t \to 0^{+} .$$

The D_{i} are tempered homogeneous distributions. We define

$$AS(\pi) = \overline{\bigcup_{i} \operatorname{supp} \hat{D}_{i}} ,$$

the closure of the union of the supports of the Fourier transforms of the D_i. $AS(\pi)$ is a union of nilpotent orbits.

Let $\lambda \in i\mathfrak{t}^*$ be integral where $\mathfrak{t}$ is the compact Cartan subalgebra. We can associate to λ a θ-stable parabolic subalgebra $q = \ell + u$ by requiring that $\ell = \mathrm{Cent}_{\mathfrak{g}}\lambda$ and $\Delta(u,\mathfrak{t}) = \{\alpha \in \Delta(\mathfrak{g},\mathfrak{t}): \langle\alpha,\lambda\rangle > 0\}$. Then λ defines a character $\lambda: q \to \mathbb{C}$. We denote by $\pi^L(\lambda)$ this representation. In [VI] chapter 6 a general construction is given which in particular associates to λ an irreducible admissible module which we call $R_q(\lambda)$. It has the following property. Fix ψ_L, a positive root system in $\Delta(\ell,\mathfrak{t})$ and denote by $\{\Theta(\psi_L,\mu)\}$ the coherent family of characters determined by the discrete series character corresponding to ψ_L.

<u>PROPOSITION 3.1</u> Let $\psi = \psi_L \cup \Delta(u)$. Then

$$\mathrm{ch}\, R_q(\lambda) = \frac{1}{|W(L \cap K)|} \sum_{w \in W(\ell)} (-1)^{\ell(w)} \Theta(\psi, w(\lambda + \rho(\psi)))$$

where $W(\ell)$ is the Weyl group of ℓ and $\rho(\psi) = \frac{1}{2} \sum_{\alpha \in \psi} \alpha$.

<u>PROOF</u>. This character identity follows from standard facts about $R_q(\lambda)$ in chapter 6 of [VI] and the following elementary identity for the character of the trivial representation π_0 (first observed by Hecht and Schmid)

$$\mathrm{ch}\, \pi_0 = \frac{1}{|W_K|} \sum_{w \in W} (-1)^{\ell(w)} \Theta(\psi, w\rho(\psi))$$

for any positive root system $\psi \subseteq \Delta(\mathfrak{g},\mathfrak{t})$. We omit the details.

We now consider the following sets

$$\mathfrak{t}^u \quad \{\lambda \in \mathfrak{t}^*: (\alpha,\lambda) = 0 \text{ for } \alpha \in \Delta(\ell)$$
$$(\alpha,\lambda) > 0 \text{ for any } \alpha \in \Delta(u)\}$$
$$\mathfrak{t}^\psi = \{\mu \in \mathfrak{t}^*: (\mu,\alpha) > 0 \text{ for } \alpha \in \psi \cup \Delta(u)\}$$

We define the following distributions for $f \in C_c^\infty(\mathfrak{g})$

$$\phi_f(\mu) = \prod_{(\alpha,\mu) > 0} (\alpha,\mu) \int_{G/T} f(x\mu x^{-1})\, dx, \quad \mu \in \mathfrak{t}^\psi$$

$$\phi_f^u(\lambda) = \prod_{(\alpha,\lambda) > 0} (\alpha,\lambda) \int_{G/L_0} f(x\lambda x^{-1})\, dx, \quad \lambda \in \mathfrak{t}^u$$

where $L_0 = \mathrm{Cent}_G\lambda$. The following properties of ϕ_f, ϕ_f^u follow from well known results of Harish - Chandra.

$$f \to \phi_f(\mu),\ \phi_f^u(\lambda) \text{ are tempered distribtuions and are Schwartz functions in } \mu \text{ and } \lambda. \tag{3.2.1}$$

Let $\omega_L = \prod_{\alpha \in \psi_L} \alpha$. Then there is a nonzero constant c such that

$$\lim_{\substack{\mu \to \lambda \\ \mu \in \mathfrak{t}^\psi}} \phi_f(\mu; \omega_L) = c\, \phi_f^u(\lambda) \tag{3.2.2}$$

(we use Harish - Chandra's notation $\partial(\omega)f(x) = f(x;\omega)$)

ϕ_f and ϕ_f^u have asymptotic expansions

$$\begin{aligned} \phi_f(\mu) &\sim \Sigma\, c_i(\mu)E_i \qquad \mu \to 0 \quad \mu \in \mathfrak{t}^\psi \\ \phi_f^u(\lambda) &\sim \Sigma\, d_j(\lambda)F_j \qquad \lambda \to 0 \quad \lambda \in \mathfrak{t}^u \end{aligned} \tag{3.2.3}$$

<u>LEMMA 3.2</u> $\overline{\cup \operatorname{supp} F_i} = \{x: x = \lim_{i \to \infty} t_i\, \mathrm{Ad}x_i\, \lambda \text{ for some } x_i \in G,\ t_i \to 0^+\}$

<u>PROOF</u>. The proof is essentially the same as the ideas in 3.7 - 3.9 in [B-VI] where ϕ_f is replaced by ϕ_f^u. We omit the details.

We denote the set in lemma 3.2 by $\overline{\mathrm{Ind}_q^{\mathfrak{g}}}(0)$. This is motivated by the complex groups case where the set in question coincides with the usual notion of induction. As the notation suggests, it can be defined for any orbit in $\ell_0 = \ell \cap \mathfrak{g}_0$. We also note that it is a somewhat finer invariant than induction from a real parabolic subalgebra. For example in $s\ell(2,\mathbb{R})$ it is always the closure of just one real nilpotent orbit instead of two. Motivated by some more examples computed for real Lie algebras we make a conjecture.

<u>3.3. CONJECTURE</u>. $\overline{\mathrm{Ind}_q^{\mathfrak{g}}}(0)$ is the closure of exactly one real nilpotent orbit. We call it $\mathrm{Ind}_q^{\mathfrak{g}}(0)$, the Richardson orbit attached to q.

PROPOSITION 3.4

$$AS(R_q(\lambda)) = \overline{\mathrm{Ind}_q^{\mathfrak{g}}}(0) .$$

PROOF. By [Ro] we can write (up to constants)

$$\theta(\psi,\mu) \sim \sum_i c_i(\mu)E_i$$

Then

$$R_q(\lambda) \sim \sum_i \Big(\sum_{w \in W(\ell)} (-1)^{\ell(w)} c_i(\lambda+\rho(\psi))\Big)E_i$$

$$= \sum_i c_i^L (\lambda+\rho(\psi))E_i .$$

Then $\hat{E}_i$ is obtained from ϕ_f (up to constants) by the formula

$$\lim_{\substack{\mu \to 0 \\ (\alpha,\mu) > 0 \\ \alpha \in \psi}} \phi_f(\mu;\, c_i^L) = \hat{E}_i(f) .$$

By (3.2.1), (3.2.2)

$$\overline{\mathrm{Ind}_q^{\mathfrak{g}}}(0) \subseteq \bigcup_{c_i^L \neq 0} \operatorname{supp} \hat{E}_i$$

To show the converse inclusion we argue as follows. Since c_j^L is $W(\ell)$ - skew invariant, $c_j^L = \omega_L\, G_j$ where G_j is $W(\ell)$ invariant. Then

$$\phi_f(\mu) = \prod_{\alpha \in \Delta(u)} (\alpha,\mu) \int_{G/L_0} \phi^L_{f^{x_1}}(\mu)\, dx_1$$

where $f^{x_1}(X) = f(x_1 X x_1^{-1})$ and ϕ^L is the function ϕ_f defined relative to ℓ. Denoting by $\tilde{G}_j$ the invariant polynomial on ℓ corresponding to G_j under the Harish - Chandra isomorphism, we obtain up to a nonzero constant,

$$\phi_f(\mu;\, c_j^L) = \prod_{\alpha \in \Delta(u)} (\alpha,\mu) \int_{G/L_0} \phi_{\partial(\tilde{G}_j)f^{x_1}}(\mu;\, \alpha(\omega_L))\, dx_1$$

Letting $\mu \to 0$ we may assume that $\mu \in \mathfrak{t}^u$ (since ϕ_f extends as a C^∞ function to the closure of $\mathfrak{t}^\psi = \{\mu \in \mathfrak{t}^*: (\mu,\alpha) > 0 ,\ \alpha \in \psi\}$). This shows that $\operatorname{supp} \hat{E}_i \subseteq \overline{\mathrm{Ind}_q^{\mathfrak{g}}}(0)$.

This proves the proposition.

4. AN EXAMPLE.

In this section we compute the cells introduced in 2.5 for $G = U(p,q)$. We assume $p \geq q$ and write $n = p + q$. We use the standard realization of $U(p,q)$ as $n \times n$ matrices that leave $\sum_{i \leq p} |x_i|^2 - \sum_{j > p} |x_j|^2$ invariant.

Representatives of the θ-stable Cartan subgroups are given by $H^r = T^r \cdot A^r \quad 0 \leq r \leq q$ where

$$T^r = \mathrm{diag}(e^{i\phi_1},\dots,e^{i\phi_{p-r}}, e^{i\phi_{p-r+1}},\dots,e^{i\phi_p}, e^{i\phi_p},\dots,e^{i\phi_{p-r+1}}, e^{i\phi_{p+r+1}} \dots e^{i\phi_n})$$

$$A^r = \mathrm{diag}(t^{x_{p-r+1}},\dots,t^{x_p}),\quad t^{x_i} = \exp[x_i(E_{p-1,p+1} + E_{p+1,p-1})]$$

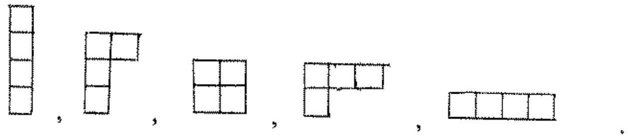
($E_{j,k}$ is the $n \times n$ matrix with 1 in the (j,k) entry and 0's everywhere else.)

Then

$$W(H) = S_{p-r} \times [(\mathbb{Z}/2\,\mathbb{Z})^r \ltimes S_r] \times S_{q-r}$$

Here S_m is the symmetric group in m letters and S_{p-r} permutes $(\phi_1,\dots,\phi_{p-r})$, S_{q-r} permutes $(\phi_{p+r+1},\dots,\phi_r)$, $(\mathbb{Z}/2\,\mathbb{Z})^r \ltimes S_r$ permutes $(\phi_{p-r+1},x_{p-r+1}),\dots(\phi_p,x_p)$ and changes the signs of the x_i . Then $\det_I = \mathrm{sign} \otimes \mathrm{trivial} \otimes \mathrm{sign}$ we call $(\mathbb{Z}/2\,\mathbb{Z})^r \ltimes S_r = W(C_r)$, the Weyl group of type C_r.

The representations of S_m are parametrized by partitions of m or Young diagrams. For example S_4 has representations



sign ... trivial.

This is well known, A convenient reference is [R].

LEMMA 4.1. Let σ be a representation of S_ℓ. Then

a) $\mathrm{Ind}_{S_\ell \times S_k}^{S_{\ell+k}} \sigma \otimes \mathrm{sign} = \bigoplus_{\sigma^1 \in \hat{S}_{\ell+k}} \sigma^1$

where σ^1 are the diagrams obtained by adding k squares to σ so that no two are added to the same row.

b) $\mathrm{Ind}_{W(C_r)}^{S_{2r}}(\mathrm{trivial}) = \bigoplus_{\sigma \in \hat{S}_{2r}} \sigma$

where σ has even rows only. The multiplicities in a) and b) are all 1.

PROOF. a) is well known. We refer to [R]. b) is proved by induction. An application of a) and a dimension computation shows that

$$\mathrm{Res}_{S_{2r-1}}^{S_{2r}} (\mathrm{Ind}_{W(C_r)}^{S_{2r}} (\mathrm{trivial})) = \mathrm{Ind}_{W(C_{r-1})}^{S_{2r-1}} (\mathrm{trivial}) \qquad (4.1.1)$$

and the right hand side is known by induction. It consists of $\sigma \in \hat{S}_{2r-1}$ with exactly one row of odd length, with multiplicity 1. We have also used the fact that the restriction of $\sigma \in \hat{S}_{2r}$ to S_{2r-1} is the sum of all $\sigma^1 \in \hat{S}_{2r-1}$ obtained from σ by deleting one corner.

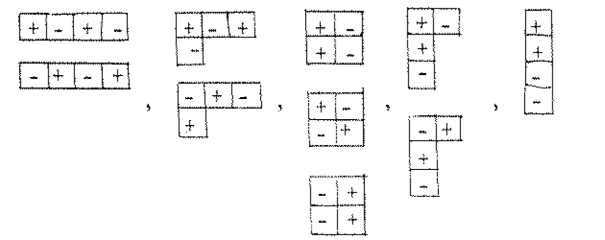
It follows that $\mathrm{Ind}_{W(C_r)}^{S_{2r}}$ (trivial) consists of σ's that have only even rows or σ's that have only two rows each of odd length. A descending induction on the length of the larger row shows that the latter cannot happen.

To compute the multiplicity of each σ we use (4.1.1) and the restriction formula.

Using lemma 4.1 and induction in stages we get the following procedure

Multiplicity of σ in F : Take p pluses and q minuses and place them alternately on each row starting with either a + or a −. The number of different ways in which this can be done is $m(\sigma,F)$. In this procedure two such "signed σ" are the same if they only differ by a permutation of equal rows.

For example, in $U(2,2)$ there are



This procedure is set up so that each representation is in a one-to-one correspondence with the conjugacy classes of nilpotent orbits in $U(p,q)$ under the adjoint action of $U(p,q)$. The next theorem is the reason for this parametrization.

THEOREM 4.2. Let $G=U(p,q)$. The cells in F are in one-to-one correspondence with the set of nilpotent orbits in $\mathfrak{g}$.

1. This correspondence is such that the representations in the cell parametrized by $O(X)$ (the orbit of $X \in \mathfrak{g}$) have $\overline{O(X)}$ as their asymptotic support.

2. To each cell one can attach a, canonically defined, θ-stable parabolic subalgebra $\mathfrak{q}$ such that $R_{\mathfrak{q}}(\lambda)$ is in the cell.

3. Each cell is an irreducible representation σ determined by the harmonic polynomial W_L (which transforms like the representation σ under W).

4. Let T_χ be the canonical invariant measure determined by $O(X)$. Then it can be normalized in such a way that

$$\lim_{\substack{\mu \to 0 \\ (\mu,\alpha) > 0 \\ \alpha \in \psi}} \phi_f(\mu; \partial(\omega_L)) = T_\chi(f) .$$

PROOF. This is more or less straightforward given the results in 2, 3, 4. We describe the procedure for obtaining q from the nilpotent orbit and omit the other details. Suppose σ has k columns. Fill each column with λ_1's , $\lambda_2 \neq \lambda_1, \ldots, \lambda_k \neq \lambda_1, \ldots, \lambda_{k-1}$. Then the λ_i's labelled + are placed in the first p coordinates of $T = H^0$, the λ_i's labelled - in the others.

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