If $\{w_n, Y_n\}$ is any other choice, then since each $Y_n(\omega)$ is one of the $X_j(\omega)$, $j = 1$, $\hat{Y}_n \geq Y_n$ and consequently $E(\hat{Y}_n) \geq E(Y_n)$. In particular, $E(\hat{Y}_n) \geq \eta_n$.

Now $E(\hat{Y}_n)$ is easily computed (see (*) below) and is given by

$$E(\hat{Y}_n) = 1 - \int_0^1 F^n(w)\, dw$$

so that

$$1 - \eta_n \geq \int_0^1 [G'(w)]^n\, dw.$$

This can also be seen analytically by noting that if G_j denotes the jth iterate of G, th

$$1 - \eta_n = \int_0^1 \frac{d}{dw} G_n(w)\, dw = \int_0^1 \prod_{j=0}^{n-1} G'(G_j(w))\, dw \geq \int_0^1 [G'(w)]^n\, dw$$

since $G(w) \geq w$. This in turn gives a sharpening of Corollary 1(b), namely,

$$\sum 1 - \eta_n = \infty \text{ provided } \int_0^1 \frac{dw}{1 - G'(w)} = \infty.$$

We conclude by asking how the best a priori choices of the w_n, namely $w_n = \eta$, corresponding expected values, η_n, compare with the best strategy using hindsight, nam

For fixed n, let $X_n^1, X_n^2, \ldots, X_n^n$ be the order statistics associated with the X's. Tha

$$X_n^1 = \min(X_1, \ldots, X_n), \quad \text{and} \quad X_n^j = \text{the } j\text{th smallest } X$$

(see **3**, Chapter 9, for a treatment of order statistics). The expectation of X_n^k is giv formula,

$$(*) \qquad E(X_n^k) = n\binom{n-1}{k-1}\int_0^1 F^{n-1}(u)\, u^{k-1}(1-u)^{n-k}\, du.$$

Consider the family of distributions $F_p(w)$ given by

$$F_p(w) = 1 - (1-w)^p, \quad p > 0.$$

The asymptotic nature of η_n for the corresponding $G_p(w)$ is known:

$$1 - \eta_n \sim \left(\frac{1 + \frac{1}{p}}{n}\right)^{1/p}.$$

A neat proof of this is given in (**4**, p. 223). Putting F_p into (*), routine calculations giv

$$(**) \qquad E(X_n^k) = n\binom{n-1}{k-1}\int_0^1 \left(1 - (1-u)^{1/p}\right) u^{k-1}(1-u)^{n-k}\, du$$

$$= 1 - \frac{\Gamma(n+1)}{\Gamma\left(n + \frac{1}{p} + 1\right)} \frac{\Gamma\left(n - k + \frac{1}{p} + 1\right)}{\Gamma(n-k+1)}.$$

Thinking of $n - k$ as fixed, say $n - k + 1 = \theta$, and using the fact that

$$\Gamma(n+z) \sim n^z\Gamma(n) \quad [\mathbf{2}, \text{p. } 212],$$

we look for k so that $1 - E(X_n^k) \sim 1 - \eta_n$ or, using (*) and (**), we try to solve

$$\frac{\Gamma\left(\theta + \frac{1}{p}\right)}{\Gamma(\theta)} = \left(\frac{p+1}{p}\right)^{1/p}.$$

re a few solutions (the last three being approximate):

$$p = 1 \text{ (uniform)}, \qquad k = n - 1,$$
$$p = 1/2, \qquad k = n - 1.50,$$
$$p = 1/3, \qquad k = n - 2.08,$$
$$p = 2, \qquad k = n - 0.73.$$

e uniform distribution, using the best a priori choices, we do surprisingly well: namely as he expectation of the second largest X; for $p = 1/3$ we do almost as well as the third X, and so on. The moral seems to be that the utility of hindsight becomes more ed as the probability of getting a large number increases.

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NOTES

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For instructions about submitting Notes for publication in this department see the inside front cover.

ANOTHER NOTE ON THE INCLUSION $L^p(\mu) \subset L^q(\mu)$

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ughout this note $(\Omega, \mathscr{A}, \mu)$ will be a positive measure space and, for each $p \in (0, \infty]$, will denote the space of all $\mathscr{A}$-measurable real functions f on Ω such that $\|f\|_p < \infty$,

$$\|f\|_p = \left(\int_\Omega |f|^p\, d\mu\right)^{1/p} \text{ for } p \in (0, \infty) \quad \text{and} \quad \|f\|_\infty = \operatorname{ess\,sup}_\Omega |f|,$$

usual we identify two functions which differ only on a set of measure zero. When endowed e metric d_p of convergence in pth mean, i.e.,

$$d_p(f, g) = \|f - g\|_p \text{ for } p \in [1, \infty] \quad \text{and} \quad d_p(f, g) = \|f - g\|_p^p \text{ for } p \in (0, 1),$$

becomes a complete metric space. We obtain a new characterization of the spaces μ) for which the inclusion $L^p(\mu) \subset L^q(\mu)$ holds. This result simplifies both the conditions proofs already given in [**1**] and [**4**].

begin with a well-known lemma.

MA 1. *Let $p, q \in [1, \infty]$. The set theoretic inclusion $L^p(\mu) \subset L^q(\mu)$ implies that the inclusion $L^p(\mu) \to L^q(\mu)$ is continuous.*

f. If $f_n \to f$ in $L^p(\mu)$, then $\{f_n\}$ has a subsequence which converges pointwise almost here to f; see [**3**], Theorem 3.12. The desired result now follows easily from the Closed

Graph Theorem; see [2], Theorem 2.15. ∎

REMARK. Lemma 1 also holds for $p, q \in (0, \infty]$, with the same proof, even though not a normed space for $0 < p < 1$.

Let $\mathscr{A}_0$ denote the collection of all sets $A \in \mathscr{A}$ with positive measure. Then we have

THEOREM 1. *The following conditions on the measure space* $(\Omega, \mathscr{A}, \mu)$ *are equivalent*

(1) $L^p(\mu) \subset L^q(\mu)$ *for some* $p, q \in (0, \infty]$ *with* $p < q$,

(2) $\inf_{E \in \mathscr{A}_0} \mu(E) > 0$,

(3) $L^p(\mu) \subset L^q(\mu)$ *for all* $p, q \in (0, \infty]$ *with* $p < q$.

Proof. (1) ⇒ (2). Since $L^p(\mu) \subset L^q(\mu)$ implies $L^{pt}(\mu) \subset L^{qt}(\mu)$ for every $t \in (0,$ … assume $p \geq 1$. Then $L^p(\mu)$ and $L^q(\mu)$ are normed spaces, and by Lemma 1 there exists constant k such that $\|f\|_q \leq k\|f\|_p$ for every $f \in L^p(\mu)$. In particular we have

$$\{\mu(E)\}^{1/q} \leq k\{\mu(E)\}^{1/p},$$

and hence $\mu(E) \geq k^{pq/(p-q)}$ for every $E \in \mathscr{A}$ with $0 < \mu(E) < \infty$. This proves (2).

(2) ⇒ (3). Let $f \in L^p(\mu)$ and let $E_n = \{|f| > n\}$, $n = 1, 2, \ldots$. By Chebyshev's … $\mu(E_n) \to 0$ as $n \to \infty$, hence, by condition (2), there is an index n_0 such that $\mu(\ldots$ $n \geq n_0$, i.e., $|f| \leq n_0$ μ-a.e. Thus $L^p(\mu) \subset L^\infty(\mu)$, and this easily implies that $L^p(\mu) \subset \ldots$ every $q \in [p, \infty]$.

(3) ⇒ (1). This is trivial. ∎

Let $\mathscr{A}_\infty$ denote the collection of all sets $A \in \mathscr{A}$ with finite measure. Then we have

THEOREM 2. *The following conditions on the measure space* $(\Omega, \mathscr{A}, \mu)$ *are equivalent*:

(1) $L^p(\mu) \supset L^q(\mu)$ *for some* $p, q \in (0, \infty)$ *with* $p < q$,

(2) $\sup_{E \in \mathscr{A}_\infty} \mu(E) < \infty$,

(3) $L^p(\mu) \supset L^q(\mu)$ *for all* $p, q \in (0, \infty)$ *with* $p < q$.

Proof. (1) ⇒ (2). As in Theorem 1, we can assume $p \geq 1$, so by Lemma 1 there is a constant k such that $\|f\|_p \leq k\|f\|_q$ for every $f \in L^q(\mu)$. It follows that

$$\mu(E) \leq k^{pq/(q-p)} \qquad \text{for every } E \in \mathscr{A}_\infty,$$

and hence condition (2) holds.

(2) ⇒ (3). Let $f \in L^q(\mu)$ and let

$$E_n = \{1/(n+1) \leq |f| < 1/n\}, \qquad n = 1, 2, \ldots.$$

Then

$$\mu(E_n) \leq (n+1)^q \int_\Omega |f|^q \, d\mu < \infty \qquad \text{for every } n = 1, 2, \ldots,$$

and hence, by condition (2), $\sum_{n=1}^\infty \mu(E_n) < \infty$, because the E_n's are pairwise disjoint. … $p < q$ we have

$$\int_\Omega |f|^p \, d\mu = \int_{\{|f| \geq 1\}} |f|^p \, d\mu + \sum_{n=1}^\infty \int_{E_n} |f|^p \, d\mu \leq \int_\Omega |f|^q \, d\mu + \sum_{n=1}^\infty \frac{1}{n^p} \mu(E_n) < \infty$$

(3) ⇒ (1). This is trivial. ∎

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SPACES WHERE ALL CONTINUITY IS UNIFORM

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…lementary topology courses normally include a proof that all continuous functions from a …pact metric space to a metric space are uniformly continuous. We abbreviated this by saying …for compact metric spaces all continuity is uniform. The aim of this note is to give several …alent conditions, which are necessary and sufficient for all continuity to be uniform. It is …to check that compactness is not such a condition, because necessity fails. …he conditions are stated formally in the following theorem.

…HEOREM. *For a metric space* (X, d), *the following conditions are equivalent*:
Every continuous function from X *to any metric space is uniformly continuous*;
every open covering of X *has a Lebesgue number*;
for every sequence (x_n) *into* X *which has no convergent subsequence, the only sequences* (x_n') *…uch that* $\lim d(x_n, x_n') = 0$ *are those which are almost equal to* (x_n), *in the sense that* $x_n = x_n'$ *…ll but a finite set of indices*;
for any infinite subset A *of* X *without accumulation points (in* X*), the infimum of the distances …en (different) points of* A *is greater than* 0.

…he following observations will help to explain how the theorem comes about and how the …l is constructed. Conditions 2 and 3 of the theorem were motivated by a careful analysis of …tandard proofs of uniform continuity on compact metric spaces. In fact, one of these proofs [4, p. 234] merely uses the property of compact metric spaces that every open covering has a …gue number, i.e., a number $\delta > 0$ such that each δ-ball is contained in a set of the covering. …property, which is precisely our Condition 2, is strictly weaker than compactness, as can be …seen by considering an infinite set with the discrete metric. It turns out, in fact, that uniform …nuity of all continuous functions is equivalent to the assertion that every open covering has a …gue number. The other proof of the uniform continuity on compact metric spaces [1; 3.16.5, …] uses the characteristic property of compact (metric) spaces that every sequence has a …rgent subsequence. It is easy to see that the proof still works if we assume the weaker …lition 3. The interesting point is that once more we have a condition—the third one of our …em—which is not only sufficient, but also necessary for all continuity to be uniform. …lition 4 is a slightly different and perhaps more suggestive version of Condition 3.

…oof. We shall prove our theorem by showing that 1 ⇒ 4 ⇒ 3 ⇒ 2 ⇒ 1.
…o prove that 1 implies 4, we will begin by assuming the existence of an infinite subset A of X …no accumulation point in X and such that the infimum of the distances between different …s of A is zero. The existence of such a set will enable us to define a continuous function from …$\mathbb{R}$, which is not uniformly continuous. The general lines for the definition of such a function …s follows:
…e first construct a locally finite sequence of balls $(B(x_n, R_n))$ such that: (i) $x_n \in A$, for all n; …ch ball $B(x_n, R_n)$ has at least one point x_n' of A distinct from x_n; (iii) the sequence (R_n) of …adii converges to 0. For each n, we then define a real function f_n with support contained in … $R_n)$ and such that $f_n(x_n) = 1$ and $f_n(x_n') = 0$. These choices can be made in such a way

that each function vanishes on all points x'_m. The sum of these f_n is the continuous function wanted to define: the fact that $(d(x_n, x'_n))$ converges to zero, and that $f(x_n) \geq 1$ and $f(x'_n)$ ensures that f is not uniformly continuous.

Let us now look into the technical details. Let $r_0 = 1$; assuming $r_i > 0$ to be defined $R_{i+1} = \min(r_i, 1/(i+1))$. By hypothesis, there exist in A two distinct points x_{i+1} and x'_{i+1} that $d(x_{i+1}, x'_{i+1}) < R_{i+1}$; we choose a positive $r_{i+1} < d(x_{i+1}, x'_{i+1})$, such that the r_{i+1} centered at x_{i+1} and x'_{i+1} intersect A precisely in their centers. Such an r_{i+1} exists, because and x'_{i+1} are not accumulation points of A. Let us now see why the choices made imply x_i, x'_i, x_j, x'_j are all different for $i \neq j$. In fact x_i, x'_i (and x_j, x'_j) were chosen distinct symmetry it is then sufficient to show that, if $i < j$, the points x_i, x_j are different. Now, if we have $R_j \leq r_i$ and so the ball $B(x_i, r_i)$ will have at least two points of A, x_j and x'_j; $B(x_i, r_i)$ has just one point of A, we have $x_i \neq x_j$. Let us prove that the sequence $(B(x_n, R_n))$ locally finite. We first remark that, for each $a \in X$, there exists an $R > 0$, such that $B(a, R)$ does not intersect A. If $a = x_{n_0}$ for some n_0, then the balls $B(x_n, R_n)$ and $B(a, R/2)$ are dis for $n > \max(n_0, 2/R)$. If no term of the sequence equals a, then the same assertion is true if take $n_0 = 1$. This concludes the proof that $(B(x_n, R_n))$ is locally finite. By defining, for n and $x \in X$,

$$f_n(x) = \max(0, 1 - d(x_n, x)/r_n),$$

we have a continuous real function f_n on X whose support $\overline{B(x_n, r_n)}$ is included in $B(x_n,$ As the sequence of these supports is locally finite, it makes sense to define $f = \Sigma f_n$ and function f is continuous. To finish the proof that 1 implies 4, we prove that f is not unifo continuous. As $d(x_n, x'_n) < R_n \leq 1/n$, it is sufficient to check that, for each n, $d(f(x_n), f($ ≥ 1. From $f_n(x_n) = 1$ it results trivially that $f(x_n) \geq 1$ (and it is easy to see that in $f(x_n) = 1$, although this is not essential to the proof). Let us now show that $f(x'_n) = 0$. For $i \in \mathbb{N}$, x_i is the only point of A in the ball $B(x_i, r_i)$. Therefore x'_n does not belong to $B($ since it is a point of A distinct from x_i as we have seen. But, as f_i is zero outside $B(x_i, r_i)$ conclude that $f_i(x'_n) = 0$, so $f(x'_n) = 0$.

To prove that (4) implies (3), let (x_n) and (x'_n) be sequences such that: (i) (x_n) ha convergent subsequence; (ii) $(d(x_n, x'_n)) \to 0$. We want to show that the two sequences are alm equal. We first remark that (x'_n) has no convergent subsequence either. Let A be the union of ranges of the two sequences. A is an infinite subset of X with no accumulation points (in Condition (4) applied to this subset A ensures the existence of an $\varepsilon > 0$, such that the dista between different points of A is greater or equal to ε. For this ε, there exists a $k \in \mathbb{N}$, such $d(x_n, x'_n) < \varepsilon$ if $n > k$. But $d(x_n, x'_n) < \varepsilon$ implies $x_n = x'_n$; so the two sequences are alm equal.

Assuming now that X satisfies Condition (3), let us prove that every open covering of X h Lebesgue number (Condition (2)). Suppose the contrary: let $(U_i)_{i \in I}$ be an open covering o with no Lebesgue number. Then, for each $n \in \mathbb{N}$, $1/n$ is not a Lebesgue number of this cove therefore there exists a ball of radius $1/n - B(x_n, 1/n)$—not included in any of the open set Let i_n be such that $x_n \in U_{i_n}$; the relation $B(x_n, 1/n) \not\subset U_{i_n}$ implies the existence in $B(x_n, 1$ of an element x'_n distinct from x_n. The sequences (x_n) and (x'_n) are not almost equal $(d(x_n, x'_n))$ converges to zero. To conclude the proof that (3) implies (2) it is sufficient to s that (x_n) has no convergent subsequence. If a were the limit of a subsequence (x_{n_k}), we c choose an open set U_j such that $a \in U_j$ and a $\delta > 0$ such that $B(a, \delta) \subset U_j$; there would $p \in \mathbb{N}$ such that $n_p > 2/\delta$ and $x_{n_p} \in B(a, \delta/2)$. Therefore we would have $B(x_{n_p}, 1/n$ $B(a, \delta) \subset U_j$, contradicting the way the x_n were chosen.

For the sake of completeness, we reproduce here the usual proof that the second condi implies the first one. Given a continuous function $f: X \to Y$ and an $\varepsilon > 0$, let $\delta > 0$ l Lebesgue number of the open covering $(f^{-1}(B(y, \varepsilon/2)))_{y \in Y}$ of X. Each δ-ball in X is includ an open set of this covering, so that the distance between the images of two points of that b less than ε; so $d(x, y) < \delta$ will imply $d(f(x), f(y)) < \varepsilon$.

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Added in proof: See the paper "Is every continuous function uniformly continuous?" by F. Snipes, recently lied in Math. Magazine (Vol. 57, 1984, 169–173), for bibliographical references on this and related subjects. of these papers state conditions similar to our Conditions 3 and 4.

UNIQUE RIGHT INVERSES ARE TWO-SIDED

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A theorem may be hard to discover, even though, once discovered, it is easy to prove. The of this note is to emphasize that completely nonrigorous (some may say nonsensical) oning is perfectly acceptable in the discovery stage, and that it may furnish clues that enable to make a good guess. Proofs can come later.

Let R be an associative ring, not necessarily commutative, with unit element 1. Recall that an ent a of R is said to be *invertible* if there exists b in R so that $ab = 1$ and $ba = 1$. The ueness of the inverse b is obvious; in fact, if $ab = 1$ and $ca = 1$, then

$$b = (ca)b = c(ab) = c.$$

sider the following two questions:

I) If $1 - xy$ is invertible, must $1 - yx$ be invertible?

II) If the answer to (I) is yes, is there a simple universal relation between these two inverses, one that holds in every ring?

Let us try to tackle this by thinking of, say, the complex numbers in place of our ring, where a ple relation exists between inverses and geometric series, namely

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

east when $|x| < 1$). It may be nonsense to talk of infinite series whose terms are members of perfectly arbitrary ring R, but never mind. Pretend that the inverse z of $1 - xy$ is given by eometric series

$$z = 1 + (xy) + (xy)^2 + (xy^3) + \cdots$$
$$= 1 + xy + xyxy + xyxyxy + \cdots.$$

we accept this, then the inverse w of $1 - yx$, if there is one, ought to be

$$w = 1 + yx + yxyx + yxyxyx + \cdots.$$

ing gone this far, we might as well do a bit of factoring (i.e., assume that the distributive laws nd to our possibly nonexistent infinite series), write

$$w = 1 + y(1 + xy + xyxy + \cdots)x.$$

observe that the series in parentheses is the postulated expansion of z. We are thus led to

$$w = 1 + yzx.$$

So far, we have proved *nothing*. But we have found a candidate for w, and can test whether it the job. Indeed,

*Partially supported by the National Science Foundation and by the William F. Vilas Trust Estate.

$$(1 - yx)w = (1 - yx)(1 + yzx)$$
$$= 1 - yx + yzx - yxyzx$$
$$= 1 - yx + y[(1 - xy)z]x,$$

which is 1, because the quantity in brackets is 1.

A closer look at this computation shows something else: we have only used the assumption z is a *right* inverse of $1 - xy$ and have deduced that w is then a *right* inverse of $1 - yx$. same is true with left in place of right, because

$$(1 + yzx)(1 - yx) = 1 - yx + y[z(1 - xy)]x.$$

Thus we get the following result, which actually does more than just answer our two questi

THEOREM 1. *If z is a right [left] inverse of $1 - xy$, then $1 + yzx$ is a right [left] inver* $1 - yx$.

As we just saw, the proof of this theorem is a total triviality. But if we had been unwillin use infinite series in a context where they may make no sense, how difficult would it have bee *discover* $1 + yzx$?

When $y = 1$, our two questions are of course pointless, but Theorem 1 tells us something then that we might not have noticed otherwise, namely:

If z is a right inverse of $1 - x$, so is $1 + zx$.

Let us now assume that $1 - x$ has a unique right inverse z. Then it follows that $1 + zx$ This implies $z(1 - x) = 1$, *so that z is also a left inverse of* $1 - x$! In other words, $1 -$ invertible. Since every element of R can be written in the form $1 - x$, we have arrived at result to which the title of this note alludes.

THEOREM 2. *The invertible elements of R are precisely those that have unique right inverses*

Of course, the same is true with left in place of right.

The following well-known fact from linear algebra is also an immediate consequen Theorem 1:

If A and B are n-by-n matrices over some field, then AB and BA have the same eigenvalue

P.S. After completing this note, I was told that the geometric series trick of finding $1 + yzx$ is described and [2]. However, no conclusions about one-sided inverses are drawn there. In the context of Banach algebras series do make sense) $1 + yzx$ occurs in an exercise on p. 259 of [4]. The referee has pointed out that Theo appears as Exercise 6 on p. 89 of [3].

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ANSWER TO PHOTO ON PAGE 465

David Blackwell.

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Telegraphic Reviews

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Education, S(15-16). Word Problems for Maxima and Minima from Computations to Equations. Stanley Bezuszka. Motivated Math Project Activity, Booklet 13. Boston College Pr, 1984, 240 pp, (P). All the max-min problems a teacher would ever want to see. Problems solved without calculus by writing computer programs to list all instances and looking for the maximum or minimum values. Emphasis on developing the algebraic equation through computation and observation of patterns. Useful as source of exercises. MW

History, S*, P, L**.** I Want To Be A Mathematician: An Automathography. Paul R. Halmos. Springer-Verlag, 1985, xvi + 421 pp, $41.50. [ISBN: 0-387-96078-3] 400 pages of unadulterated Halmos on teaching ("The hardest part...is to keep your mouth shut"), on scholarship ("the most important attribute of a genuine professional mathematician"), on writing ("I like words more than numbers...I want to say that as an indication of mathematical ability, liking words is better than being good at calculus"), on research ("The greatest kind of step forward is the illuminating central example from which it is easy to get insight into all the surrounding sweeping generalities"), and on himself: "My greatest strength as a mathematician is the ability to see when two things are the 'same'." LAS

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Logic, S(16-18), P*, L.** Cantorian Set Theory and Limitation of Size. Michael Hallett. Logic Guides, No. 10. Clarendon Pr, 1984, xxii + 343 pp, $32.50. [ISBN: 0-19-853179-6] A comprehensive study of the impact that Cantor's metaphysical, philosophical, and theological views had on his sub-