Math 6240, Lie Groups and Lie Algebras I

August 20, 2018

Exercise. Let

$$G = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \mid a, b \in \mathbb{R}, c \in \mathbb{R}/\mathbb{Z} \right\} .$$

Verify that G is indeed a Lie group. Then show that there does not exist an injective homorphism $G \hookrightarrow \operatorname{GL}(n, \mathbb{R})$ for any n. This shows that there are natural examples of "nonlinear" Lie groups.