MATH 6210: WARM UP
due August 27

1. For $n \in \mathbb{N}$, let $a_n = (-1)^n/n$.

    (a) Prove or disprove: there exists a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\sum_{n=1}^{\infty} a_{f(n)}$ diverges.

    (b) Fix any $c \in \mathbb{R}$. Prove or disprove: there exists a bijection $f : \mathbb{N} \to \mathbb{N}$ such that

$$\sum_{n=1}^{\infty} a_{f(n)} = c.$$

2. [Rudin, Chapter 1, Problem 1] Recall that a σ-algebra over a set X is a nonempty collection of subsets of X which is closed under taking complements and countable unions of its members. Does there exist an infinite σ-algebra with only countably many members?