MATH 6210: ADDENDUM TO PROBLEM SET #1

due September 17th

Let $(X, \mathcal{M})$ be a measurable space equipped with a positive measure μ such that $\mu(X) < \infty$. Let $f : X \to X$ be a measure preserving transformation of X. That is, suppose that whenever $E \in \mathcal{M}$, then so is $f^{-1}(E)$; and, moreover, $\mu(E) = \mu(f^{-1}(E))$ in this case. Fix $A \in \mathcal{M}$ and assume $\mu(A) > 0$.

(a) Prove that some point of A returns to A. More precisely prove that there exists $x \in A$ and $n \in \mathbb{N}$ such that $f^n(x) \in A$.

(b) Prove that almost every element of A returns to A. More precisely prove that

$$\mu\left(\{x \in A \mid f^n(x) \notin A \text{ for all } n\}\right) = 0.$$

(c) Prove that almost every element A returns to A *infinitely often*. More precisely prove that

$$\mu\left(\{x \in A \mid \text{there exists } N \text{ such that } f^n(x) \notin A \text{ for all } n > N\}\right) = 0.$$