

PRACTICE EXAM #1
September 21, 2001

There are six questions on the practice exam. Calculators are allowed.

1 True or false:

- (1) $e^{x+y} = e^x + e^y$
- (2) $\ln(x^2) = 2 \ln(x)$
- (3) $\ln(e^{\ln x}) = \ln x$
- (4) $e^{(x^2)} = e^{2x}$
- (5) $\sqrt{e^x} = e^{\sqrt{x}}$
- (6) A system of two linear equations in two variables always has at least one solution.
- (7) If $\lim_{x \rightarrow c} f(x)$ exists, then $\lim_{c \rightarrow c^+} f(x)$ exists.
- (8) If $f(x)$ is continuous at $x = c$, then $f'(c)$ exists.
- (9) If $\lim_{x \rightarrow c} f(x)$ exists, then $f(c)$ exists.
- (10) Assume f and g are differentiable functions. Then the derivative of $f(x) + g(x)$ is $f'(x) + g'(x)$.

2. Solve the following system of equations. You may wish to use the method of Gauss-Jordan elimination.

$$\begin{aligned}x + y - z &= 0 \\x + 2y - 3z &= -5 \\2x - y - 13z &= 17\end{aligned}$$

3. Compute $f'(x)$ for the following functions $f(x)$

- (1) $f(x) = 3x^3 - 2x + 1$
- (2) $f(x) = (2x^2 + 1)^5$
- (3) $f(x) = e^{x^2+2x}$
- (4) $f(x) = \frac{x+1}{x^2+3}$
- (5) $f(x) = \ln\left(\frac{\sqrt[4]{x^3}}{\sqrt{2x+1}}\right)$

4. Find the equation of the tangent line to $f(x) = 3x^3 - 2x + 1$ at the point $(1, 2)$.

5. Determine if the following limits exist. Compute the limits if they do.

- (1) $\lim_{x \rightarrow 4} (3x^2 + x + 3)$
- (2) $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$
- (3) $\lim_{x \rightarrow 5} \frac{x^2 - 25}{(x - 5)^2}$
- (4) $\lim_{x \rightarrow 2^+} (e^{1/(2-x)})$
- (5) $\lim_{x \rightarrow 2^-} (e^{1/(2-x)})$

6. Suppose

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 2x^2 - 1 & \text{if } x \geq 1 \end{cases}$$

- (1) Sketch the graph of $f(x)$.
- (2) Compute $\lim_{x \rightarrow -1} f(x)$.
- (3) Does $\lim_{x \rightarrow 0} f(x)$ exist? If so, compute it.
- (4) Does $\lim_{x \rightarrow 1} f(x)$ exist? If so, compute it.
- (5) Is $f(x)$ continuous at $x = 0$?
- (6) Is $f(x)$ differentiable at $x = 0$?
- (7) Is $f(x)$ continuous at $x = 1$?
- (8) Is $f(x)$ differentiable at $x = 1$?
- (9) Is $f(x)$ continuous at $x = -1$?
- (10) Is $f(x)$ differentiable at $x = -1$?

ANSWERS TO PRACTICE EXAM #1

1.

- (1) F
- (2) T
- (3) T
- (4) F
- (5) F
- (6) F
- (7) T
- (8) F
- (9) F
- (10) T

2. $x = 15, y = -13, z = 2$

3.

- (1) $9x^2 - 2$
- (2) $20x(2x^2 + 1)^5$
- (3) $(2x + 2)e^{x^2+2x}$
- (4) $\frac{-x^2-2x+3}{(x^2+3)^2}$
- (5) $\frac{3}{4x} + \frac{1}{2x+1}$

4. $y = 7x - 5$

5.

- (1) 55
- (2) 10
- (3) DNE
- (4) 0
- (5) DNE

6.

- (1) You can do this.
- (2) 2
- (3) DNE
- (4) limit exists and equals 1
- (5) No
- (6) No
- (7) Yes
- (8) No
- (9) Yes
- (10) Yes