Exam 2
November 6, 2001

There are six questions on the exam. Calculators are allowed[1].

1 True or false (10 points):

(a) If $f''(c) = 0$, the f has a horizontal point of inflection (plateau) at c.

(b) If $f'(c) = f''(c) = 0$, then f has a horizontal point of inflection (plateau) at c.

(c) All continuous functions have at least one critical point.

(d) Suppose f satisfies the differential equation $\frac{df}{dx} = F(x)$. Then $f(x) = \int F(x)dx$.

(e) If profits are maximized at the sale of x units, then $\overline{MR}(x) = \overline{MC}(x)$.

2. (10 points) Find the general solution to the differential equation
$$\frac{df}{dx} = xe^{x^2}.$$

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[1] You will receive zero credit on problems 2–6 without showing your work.

3. (20 points) Compute the following indefinite integrals:

(a) $\int 2x(x^2 + 4)^5 dx$

(b) $\int \frac{\ln|x|}{x} dx$

4. (25 points) Consider the function
$$f(x) = (x - 1)^3 + (x - 2)^2 + 2x^2 - 2x^3/3.$$
(a) Find the critical values of $f(x)$.

(b) For each critical value determine whether it is a minimum, a maximum, or a horizontal point of inflection (plateau).

(c) Determine the values of x (if any) for which $f(x)$ is concave up.

(d) Determine the points of inflection (if any) for $f(x)$.

(f) Sketch the graph of $f(x)$.

5. (15 points) The revenue for the sale of x units of a certain software product is observed to obey the following differential equation:
$$\frac{1}{2x}\frac{dR}{dx} = e^{-R}.$$
Find the general solution of this differential equation. Then use the fact that R is a revenue function to eliminate the constant of ambiguity in the general solution.

6. (20 points) If the daily demand for a product is given by the function
$$p = 1000 - 25q - q^2/3,$$
and the daily supply before taxation is given by
$$p = 200 + 10q,$$
find the tax per item that maximizes total tax revenue. Find the maximum total tax revenue.