Name: ____________________ Secret Code: ____________________

Exam #1
September 27, 2001

There are seven questions on the exam. Check that you have all five pages!
Calculators are allowed.

1. (10 points; 2 points each) True or false:

   a. $\ln(x+y) = \ln(x) + \ln(y)$

   b. $e^{(\ln(e^x))} = e^x$

   c. $\ln(\sqrt{x}) = \sqrt{\ln(x)}$

   d. If two lines have the same slope, they are parallel.

   e. If $\lim_{x \to c} f(x)$ exists, then $f'(c)$ exists.

2. (10 points; 5 points each) Determine if the following limits exist. If so, compute them.

   a. $\lim_{x \to 3} \left(\frac{x^2-2x-3}{x-3} \right)$

   b. $\lim_{x \to 3} \left(\frac{x^2-2x-3}{\sqrt{(x-3)^3}} \right)$

3a. (10 points) Compute $f'(x)$ if

$$f(x) = 4x^3 - 5x^2 + 3x - 4.$$

b. (10 points) Find the tangent line to the graph of $y = f(x)$ at the point $x = 1$.

4. (10 points each) Compute $f'(x)$ for the following functions $f(x)$. For (a), express your final answer without negative or fractional exponents.

(a) $f(x) = \left(\frac{1}{\sqrt[4]{(x^2+1)^3}}\right)$

(b) $f(x) = e^{3x+2} \ln\left((x^2+1)^{10}\right)$

5. (20 points; 4 points each) Suppose

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ -x^2 - 1 & \text{if } x \geq 1 \end{cases}.$$

a. Sketch the graph of $f(x)$.

b. At what values of x is $f(x)$ **not** continuous?

c. Does the left-hand limit $\lim_{x \to 1^-} f(x)$ exist? If so, what is its value?

d. At what values of x is $f(x)$ **not** differentiable?

e. Compute $f'(2)$.

6. (10 points) Use implicit differentiation to find $\frac{dy}{dx}$ if

$$ye^x + y^2 = 0.$$

7. (10 points) A ball is thrown in the air from the 10th floor of an apartment building. Its height in feet (as a function of time in seconds) is observed to be

$$h(t) = 100 + 120t - 30t^2.$$

Recall that the velocity of the ball is given by $h'(t)$. Find the velocity as a function of time. At what time does the ball attain its maximum height?

SOLUTIONS TO EXAM #1
September 27, 2001

1.

(a) F ($\ln(xy) = \ln(x) + \ln(y) \neq \ln(x+y)$)
(b) T
(c) F ($\ln(\sqrt{x}) = (1/2)\ln(x) \neq \sqrt{\ln(x)}$)
(d) T
(e) F

2.

(a)
$$\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+1)}{x+3} = \lim_{x\to 3} x+1 = 4.$$

(b)
$$\lim_{x\to 3} \frac{x^2-2x-3}{\sqrt{(x-3)^3}} = \lim_{x\to 3} \frac{(x-3)(x+1)}{(x-3)^{3/2}} = \lim_{x\to 3} \frac{x+1}{(x-3)^{1/2}} = DNE,$$

Since the numerator is a polynomial with nonzero value at 3, and the denominator is zero at 3.

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(a) $f'(x) = 12x^2 - 10x + 3$.

(b) The slope of the tangent line at $x = 1$ is $f'(1) = 5$, and so $y = 5x + b$. The line passes through the point $(1, f(1)) = (1, -2)$, and we can use this point to solve $-2 = 5 + b$ for $b = -7$ to conclude the tangent line at $x = 1$ has equation $y = 5x - 7$.

4.

(a) $f(x) = (x^2+1)^{-3/4}$, so use the generalized power rule to compute
$$f'(x) = (x^2+1)^{-7/4}(2x) = \frac{2x}{\sqrt[4]{(x^2+1)^7}}.$$

(b) Use the product rule with $u(x) = e^{3x+2}$ and $v(x) = \ln\big((x^2+1)^{10}\big) = 10\ln(x^2+1)$. Compute $u'(x) = 3e^{3x+2}$ and $v'(x) = 20x/(x^2+1)$. So
$$f'(x) = \frac{20xe^{3x+2}}{(x^2+1)} + \frac{30e^{3x+2}}{\ln(x^2+1)}.$$

5.

(b) $x = 1$
(c) limit exists, equals 1
(d) $x = 0, 1$
(e) For $x \geq 1$, $f'(x) = -2x$; so f'(2) = -4.

6. Take $\frac{d}{dx}$ of both sizes, and use the product rule to compute the term $\frac{d}{dx}(ye^x)$ to obtain
$$ye^x + e^x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0.$$

Solve to get

$$\frac{dy}{dx} = -\frac{ye^x}{e^x + 2y}.$$

7. The velocity is $h'(t) = 120 - 60t$. The ball peaks out at its max height when its velocity is zero, i.e. when $0 = 120 - 60t$, so $t = 2$.