Quiz #4
Time: 10 minutes

Show all work. Check your answers.

Consider the function \( f(x) = \frac{x+1}{2x-3} \), and denote \( \Gamma \) its graph. (a) Find the domain of \( f \) and the vertical asymptotes of \( \Gamma \). (b) Find the horizontal asymptotes of \( \Gamma \). (c) Determine the position of \( \Gamma \) relative to its asymptotes (for horizontal ones: above/below, for vertical ones: does the graph go up or down to the left/right of the asymptote?) (d) Sketch the graph \( \Gamma \).

(a) Domain: all reals except \( \frac{3}{2} \) (where denominator is 0).

Since numerator is not 0 at \( x = \frac{3}{2} \), \( \Gamma \) has a vertical asymptote \( (x = \frac{3}{2}) \).

(b) For the horizontal asymptote, look at highest-degree terms: \( \frac{x}{2x} = \frac{1}{2} \)

so \( \Gamma \) has a horizontal asymptote \( (y = \frac{1}{2}) \).

(c) When \( x \) is very large (and positive) \( x+1 > \frac{1}{\frac{3}{2}} \).

so \( f(x) > \frac{1}{2} \): \( \Gamma \) is above \( (y = \frac{1}{2}) \)

* When \( x \) is very large (and negative), still: \( x+1 > \frac{1}{\frac{3}{2}} \)

but now \( 2x-3 < 0 \) so divide inequality by negative number to get:

* When \( x \) is slightly larger than \( \frac{3}{2} \)

\( 2x-3 \) is a (small) positive number:

so \( f(x) < \frac{1}{2} \) (and \( \Gamma \) is below \( y = \frac{1}{2} \))

* When \( x \) is slightly smaller than \( \frac{3}{2} \) ("graph shoots up")

\( 2x-3 \) is a (small) negative number, and \( x+1 \) is also positive

so \( f(x) < 0 \) to the left of vertical asymptote ("graph shoots down")

(d) If you care, \( \Gamma \) is a hyperbola