University of Utah
Math 2210, Fall 2008
Name: Key

Quiz # 4

Time: 10 minutes

Part 1: (10 points) Find the maximum and minimum values of the function $f(x, y) = 2x + 4y$ on the circle $x^2 + y^2 = 5$.

Use Lagrange multipliers. Let $g(x,y) = x^2 + y^2 - 5$.

Then: $\vec{\nabla} f(x,y) = \langle 2, 4 \rangle$ and $\vec{\nabla} g(x,y) = \langle 2x, 2y \rangle$.

This leads to the equations: $\begin{cases} 2x = 2\lambda \\ 2y = 4\lambda \\ x^2 + y^2 = 5 \end{cases}$ (where λ is the Lagrange multiplier). $\Rightarrow y = 2x$ and $x^2 + (2x)^2 = 5$.

The solutions are $(x,y) = (1,2)$ or $(-1,-2)$.

$f(1,2) = 10$ is the maximum of f on the circle,

$f(-1,-2) = -10$ is the minimum.

Part 2: (10 points) Evaluate the integral:

$$I = \int_0^1 \int_1^2 (x + 2y) dx dy.$$

Start by evaluating the inside integral:

$$\int_1^2 (x + 2y) dx = \left[\frac{x^2}{2} + 2yx \right]_1^2 = 2 + 4y - \left(\frac{1}{2} + 2y \right) = 2y + \frac{3}{2}$$

Then: $I = \int_0^1 \left(2y + \frac{3}{2} \right) dy = \left[y^2 + \frac{3}{2} y \right]_0^1 = 1 + \frac{3}{2} = \frac{5}{2}.$