#20. Straightforward.

#21. Follows from definitions.

#22: Use #21 and definitions (separate cases when f or g is $\pm\infty$)
Note that (b) doesn't mention the indeterminate form "$0\times\infty$" because Royden adopts the convention that $0\times\infty = 0$ (p. 36).

#24: Use hint. From the definition of a measurable function, if f is measurable then $f^{-1}(O)$ is measurable for any open interval O. Moreover (hint), "the class of sets E for which $f^{-1}(E)$ is measurable is a σ-algebra" (indeed: $f^{-1}(\bigcup A_\alpha) = \bigcup f^{-1}(A_\alpha)$, $f^{-1}(\bigcap A_\alpha) = \bigcap f^{-1}(A_\alpha)$)
Therefore this class of sets contains all Borel sets.

#25: Let $\alpha \in \mathbb{R}$. Then: $\{x \mid g\circ f(x) < \alpha\} = f^{-1}(\underbrace{g^{-1}((-\infty,\alpha))}_{\text{open because } g \text{ continuous}})$ is measurable.
measurable by #24 (open $\Rightarrow$ Borel).

#28. From Problem 2.48 we know that the Cantor ternary function f_1 ~~satisfies~~ is continuous, monotone on $[0,1]$, constant on each connected component of the complement of the Cantor set C, and maps C onto $[0,1]$. Let $f(x) = f_1(x) + x$.

(a) f is continuous and increasing (because f_1 is nondecreasing, x is increasing), so f is a homeo. $[0,1] \to [0,2]$

(b) Let $F = f(C) = \{f_1(x) + x \mid x \in C\}$.
Recall that f_1 is constant on each component of $[0,1] - C$, which has measure 1 ($m(C) = 0$).
Therefore, if I is one of these open intervals, $f(I)$ is a translate of I, in particular has same measure as I. Therefore: $m(f([0,1]-C)) = 1$, and $m(f(C)) = 1$.

(c) Let $g = f^{-1}$; then $g(F) = C$. Now F has measure >0, so it contains a non-measurable set B (in the same way that we saw a non-measurable set $\subset [0,1]$). Consider $A = g(B)$.
Then A is measurable (because $\subset C$ of measure 0), but $g^{-1}(A) = B$ is not.

(d) Take g as above and $h = \chi_A$. Then: $\{x \mid h\circ g(x) > 0\} = \{x \mid g(x) \in A\} = g^{-1}(A)$ is not measurable, so $h\circ g$ is not measurable (⚠ See problem 25: the order of composition matters!)
Note that: f_1, f_2 measurable $\not\Rightarrow$ $f_1 \circ f_2$ measurable.

(e) Take A again, A is measurable but not a Borel set (or else, problem 24 would tell us that $g^{-1}(A)$ is measurable)

#29 Take $E = \mathbb{R}$, $f_n(x) = \frac{x}{n}$. Then the conclusion of prop. 23 doesn't hold.

#1, ch. 4: (a) Any step function h s.t. $h \le f$ is identically 0 (any open interval contains an irrational) and any step function g s.t. $g \ge f$ is identically 1 (..... rational).
Therefore: $R\int_a^b f = b-a$ and $R\int_a^b f = 0$, so f is not Riemann integrable. (Of course, it is Lebesgue integrable)

(b) $\mathbb{Q}\cap[a,b]$ is countable; let $(r_n)_{n\ge1}$ be an enumeration of $\mathbb{Q}\cap[a,b]$. Let $f_n = \chi_{\{r_1,\dots,r_n\}}$.
Then f_n increases to f, and each f_n is Riemann integrable. Cannot change the order...