#20: Straightforward.

#21: Follows from definitions.

#22: Use #21 and definitions (separate cases when f or g is \( \pm \infty \)).

Note that (4) doesn't mention the indeterminate form "\( \infty \times \infty \)" because Royden adopts the convention that \( \infty \times \infty = 0 \) (p. 36).

#24: Use hint. From the definition of a measurable function, if \( f \) is measurable then \( f^{-1}(O) \) is measurable for any open interval \( O \). Therefore (hint), the class of sets for which \( f^{-1}(E) \) is measurable is a \( \sigma \)-algebra. Indeed: 
\[
\begin{align*}
 f^{-1}(\{0\}) &= f^{-1}(U \cap \{0\}) \\
 &= f^{-1}(U) \cap f^{-1}(\{0\})
\end{align*}
\]

Therefore this class of sets contains all Borel sets.

#25: Let \( E \subseteq \mathbb{R} \). Then: 
\[
\{ x \mid g(f(x)) < x \} = f^{-1}(g^{-1}((\infty, \infty)) \text{ is measurable.}
\]

#28: From Problem 2.48 we know that open becomes continuous measurable by #24 (open \( \Rightarrow \) Borel).

Recall that \( f \) is a continuous linear function on \( \mathbb{R} \) (counterexample is Cantor ternary function \( f \)).

Therefore if \( I \) is one of these open intervals, \( f[I] \) is a translate of \( I \) with the same measure as \( I \). Therefore: \( m(f[I] - C) = 1 \) and \( m(f[I]) = 1 \).

Let \( g = f^{-1} \). Then: \( g(F) = C \). Now \( F \) has measure \( >0 \), so it contains a non-measurable set \( B \) (in the same way that we saw a non-measurable set \( C \)). Consider \( A = g(B) \).

Then \( A \) is measurable (because \( C \) is of measure 0), but \( g^{-1}(A) = B \) is not.

Take \( A = \{ x \mid f(x) > 0 \} \). Then: \( A = \{ x \mid g(x) \in A \} \) is measurable, so \( g(x) > 0 \) is measurable! (\( \Delta \) see problem 25: the order of composition matters.)

(c) Take \( A \) again, \( A \) is measurable but not a Borel set (or else problem 24 would tell us that \( f^{-1}(A) \) is measurable; 

#29: Take \( F : \mathbb{R} \to \mathbb{R} \), \( f_n(x) = \frac{1}{x^n} \). Then the conclusion of prop. 23 doesn't hold.

#41: (a) Any step function \( h \) is \( \mathcal{B} \)-measurable.

Therefore: \( g \circ f \). \( f \) is identically \( 1 \) (any open interval contains an irrational) and any step function \( g \) is identically \( 1 \) (any open interval contains a rational).

(b) \( g \circ f \) is integrable; \( \int g \circ f \, dx = m(\mathbb{R}) \), \( f \) is an enumeration of \( \{0, 1\} \). \( f \) is not Riemann integrable! Of course it is Lebesgue integrable.

(c) \( f \) is increasing to \( f \) and each \( f_n \) is Riemann integrable. Cannot change the order...