$\boxed{\#1\ p.288}$: If $x \in E \cap E^{\perp}$, then in particular $\langle x, x\rangle = 0$, so $x = 0$.

$\boxed{\#2}$: Let $y \in E^{\perp}$. Since E is dense, there exists a sequence (x_n) in E converging to y. But $\langle x_n, y\rangle = 0$ for all n ($x_n \in E$ and $y \in E^{\perp}$), so $\langle y, y\rangle = 0$ (by continuity of $\langle \cdot, y\rangle$) and $y = 0$.

$\boxed{\#3}$: This is just rephrasing the definition

(Note that, if $Y \cap Z \neq \{0\}$, then the decomposition $x = y + z$ with $y \in Y$ and $z \in Z$ is not unique: indeed, if $w \in Y \cap Z \setminus \{0\}$, then $x = (y+w) + (z-w)$ with $y+w \in Y$, $z-w \in Z$).

$\boxed{\#4}$: Seen in class: $\|x+y\|^2 = \langle x+y, x+y\rangle = \|x\|^2 + \|y\|^2 + 2\langle x, y\rangle$ (or $2\mathrm{Re}\langle x, y\rangle$ if $\mathbb{C}$)

$\boxed{\#5}$: We know that if Y is a closed linear subspace of X, then $X = Y \oplus Y^{\perp}$.
Analogously, $X = Y^{\perp} \oplus Y^{\perp\perp}$ ($Y^{\perp}$ is also a closed linear subspace). Therefore $Y = Y^{\perp\perp}$.

$\boxed{\#6}$: Again, continuity of $\langle \cdot, \cdot\rangle$.

$\boxed{\#7}$ $= \#5 + \#6 +$ bilinearity of $\langle \cdot, \cdot\rangle$

$\boxed{\#8}$ Again, if Y is a closed linear subspace of X, we've seen that $X = Y \oplus Y^{\perp}$ and that this decomposition is given by: $x = \underbrace{P_Y(x)}_{\in Y} + \underbrace{(x - P_Y(x))}_{\in Y^{\perp}}$ for all x.

$\boxed{\#1\ p.292}$: Pythagorean Theorem + induction

$\boxed{\#2}$ Take a limit in #1.

$\boxed{\#3}$ If $x = (x_1, x_2, \dots, x_n, \dots) \in l^2$, then $\langle x, e_k\rangle = x_k$.

$\boxed{\#4}$ By Bessel's Inequality $\left(\sum_{k=1}^{\infty} |\langle x, u_k\rangle|^2 \leq \|x\|^2\right)$ we know that the series $\sum |\langle x, u_k\rangle|^2$ converges. Therefore $|\langle x, u_k\rangle|^2 \xrightarrow[k\infty]{} 0$, so $\langle x, u_k\rangle \longrightarrow 0$.