

Katz 7.25

$$\text{Solve } \begin{cases} N \equiv 0 \pmod{3} \\ N \equiv 1 \pmod{4} \end{cases}$$

Answer:  $N \equiv 9 \pmod{12}$

(or if just one such  $N$  is given)

(Systematically: to solve  $3x = 4y + 1$  in  $\mathbb{Z}$ , we use Euclid's algorithm, but clearly  $x = -1, y = -1$  works  $\leadsto N = -3$  is a solution, and all solutions differ from this one by a multiple of ~~12~~  $12 = 3 \cdot 4$ ).

Katz 7.26

$$\text{Solve } \begin{cases} N \equiv 0 \pmod{11} \\ N \equiv 0 \pmod{5} \\ N \equiv 0 \pmod{7} \\ N \equiv 4 \pmod{9} \\ N \equiv 6 \pmod{8} \end{cases}$$

Note that the first 3 equations are equivalent to  $N \equiv 0 \pmod{5 \cdot 7 \cdot 11}$  so we instead treat the system

$$\begin{cases} N \equiv 0 \pmod{385} \\ N \equiv 4 \pmod{9} \\ N \equiv 6 \pmod{8} \end{cases}$$

Find  $N_1$  such that  $\begin{cases} N_1 \equiv 1 \pmod{9} \\ 0 \pmod{8} \\ 0 \pmod{385} \end{cases}$  by solving  $\frac{385 \cdot 8 \cdot x_1}{9} \equiv 1 \pmod{9}$ ,  
 ie  $2x_1 \equiv 1 \pmod{9}$ . So  $x_1 \equiv 5 \pmod{9}$

Find  $N_2$  s.t.  $\begin{cases} N_2 \equiv 1 \pmod{8} \\ 0 \pmod{9} \\ 0 \pmod{385} \end{cases}$  by solving  $\frac{385 \cdot 9 \cdot x_2}{8} \equiv 1 \pmod{8}$ ,  
 ie  $x_2 \equiv 1 \pmod{8}$

Then  $N_1 = 385 \cdot 8 \cdot 5$ , and  $N_2 = 385 \cdot 9 \cdot 1$ , and  $N = 4N_1 + 6N_2 = 82,390$  works,

ie  $N \equiv 26,950 \pmod{27,720}$  gives all solutions  
 $= 5 \cdot 7 \cdot 8 \cdot 9 \cdot 11$

(again, just giving one answer is OK)

(3010 PSet 3 ctd)