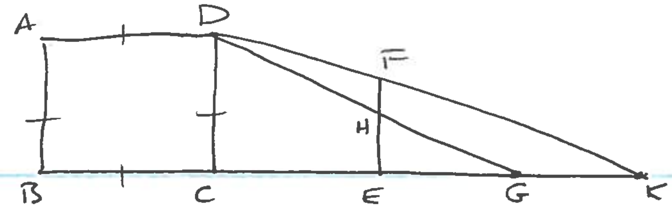
Katz 7.18



Given: $EF = 10$

$EG = 5$

$EH = 3\frac{31}{40} = \frac{151}{40}$

$EK = \frac{40}{3}$

Find DC and EC. $\triangle EHG$ is similar to $\triangle CDG$

$\triangle EFK$ is similar to $\triangle CDK$

$$\Longrightarrow \quad \frac{CG}{EG} = \frac{CD}{EH} \quad \text{and} \quad \frac{CK}{EK} = \frac{CD}{EF}, \text{ ie}$$

$$\frac{CE+5}{5} = \frac{CD}{151/40} \quad \text{and} \quad \frac{CE + 40/3}{40/3} = \frac{CD}{10}$$

Let $x = CE$, $y = CD$, so $151(x+5) = 200y$ and $3x + 40 = 4y$

$\Rightarrow 151x + 151\cdot 5 = 150x + 50\cdot 40 \Rightarrow x = 2000 - 755$

$\Rightarrow \boxed{EC = 1245\text{ ft}}$

and then $y = (3\cdot 1245 + 40)/4 = 943.75 \Rightarrow \boxed{CD = 943.75\text{ ft}}$

Katz 7.23 $y^4 = 279{,}841$. Solve by Qin's procedure.

Note $20^4 < 279{,}841 < 30^4$, so $y = 20 + x$ with $0 \le x < 10$

Use synthetic division to write the equation in terms of x

$$\begin{array}{r|rrrrr}
20 & 1 & 0 & 0 & 0 & -279{,}841 \\
 & & 20 & 400 & 8000 & 160000 \\ \hline
20 & 1 & 20 & 400 & 8000 & -119841 \\
 & & 20 & 800 & 24000 & \\ \hline
20 & 1 & 40 & 1200 & 32000 = \binom{4}{3}\cdot 20^3 & \\
 & & 20 & 1200 & & \\ \hline
20 & 1 & 60 & 2400 = \binom{4}{2}\cdot 20^2 & & \\
 & & 20 & & & \\ \hline
20 & 1 & 80 = \binom{4}{1}\cdot 20^1 & & & \\
 & 1 = \binom{4}{0}\cdot 20^0 & & & &
\end{array}$$

so x satisfies $0 = x^4 + 80x^3 + 2400x^2 + 32{,}000x - 119{,}841$ and $x = 3$ works. Thus the original equation has $\boxed{y = 23}$ as a solution

These equalities, and $-119841 = \binom{4}{4}\cdot 20^4 - 279{,}841$, give the relationship with Pascal's triangle (via the binomial expansion of $(x+20)^4 = 279{,}841$).