3010 PSet 2

(1) $\gcd(95,133)=19$

Justification:

$133 = 95\cdot 1 + 38$

$95 = 38\cdot 2 + 19$

$38 = 19\cdot 2 + 0$

(2) Solve (in $\mathbb{Z}$) $49x+33y=1$.

Answer: $(x,y) = (-2+33t, 3-49t)$ for any integer t: give full credit for any single solution.

$49 = 33\cdot 1 + 16$

$33 = 16\cdot 2 + 1$

$\Rightarrow \quad 1 = 33 - 16\cdot 2 = 33 - [49 - 33\cdot 1]\cdot 2 = 3\cdot 33 - 2\cdot 49$

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(3) Katz 7.6

Answer $\frac{15}{74} = .2\overline{027}$ days

Let t = time in days

In time t, R1 fills $3t$ reservoirs

R2 " t

R3 " $2/5\,t$

R4 " $1/3\,t$

R5 " $1/5\,t$

all channels fill $74/15\,t$ reservoirs $\Rightarrow$ the reservoir is full in $15/74$ days

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(4) Katz 7.8 Let S_{2n} = area of regular $(2n)$-gon inscribed in a circle of radius 1. Let c_n = side of regular n-gon, a_n = apothem of regular n-gon.

Then
$$\begin{cases} S_{2n} = \frac{1}{2} n c_n \\ c_{2n}^2 = (1-a_n)^2 + \left(\frac{c_n}{2}\right)^2 \\ 1 = \left(\frac{c_n}{2}\right)^2 + a_n^2 \end{cases}$$

Starting from $c_6 = 1$, $a_6 = \frac{\sqrt{3}}{2}$, this allows inductive calculation of $S_{12}, S_{24}, S_{48}, S_{96}, S_{192}$

Answer: $S_{12} = 3$

~~$c_{12} \approx .5176382$, $a_{12} \approx$~~

(It doesn't matter how many digits of decimal approx they give)

$S_{24} = 6\cdot c_{12} \approx 3.105829$

$S_{48} = 12\cdot c_{24} \approx 3.132629$

$S_{96} = 24\cdot c_{48} \approx 3.139350$

$S_{192} = 48\cdot c_{96} \approx 3.141032$

(Exact answers given by
$c_{12} = \sqrt{2-\sqrt{3}}$
$c_{24} = \sqrt{2-\sqrt{2+\sqrt{3}}}$
$c_{48} = \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$
$c_{96} = \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}}$)

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(5) Katz 7.9 Volume of double-box-lid

$= \int_{-r}^{r} (\text{cross-sectional area at height } h)\,dh$

$= 2\cdot \int_0^r 4(r^2-h^2)\,dh = 8r^3 - \frac{8r^3}{3}$

$= \boxed{\frac{16r^3}{3}}$

this cross-sectional area was explained in class, so they don't have to justify this step