6 Parts (1) (3 each)

(a) \[ 1 - \frac{11}{19} \cdot 1 = \frac{8}{19} \] (I have abbreviated the successive subtractions as "division". It would also be ok to write, e.g., \( \frac{11}{19} - \frac{3}{19} = \frac{8}{19} \), \( \frac{11}{19} - \frac{11}{19} = \frac{2}{19} \) in line 3.)

\[ \frac{2}{19} - \frac{11}{19} \cdot 2 = 0 \]

\[ \frac{3}{5} - \frac{1}{3} \cdot 1 = \frac{4}{15} \]
\[ \frac{1}{3} - \frac{4}{15} \cdot 1 = \frac{1}{15} \]
\[ \frac{4}{15} - \frac{1}{15} \cdot 4 = 0. \]

(b) \[ \sqrt{3} - 1.1 = \sqrt{3} - 1 \]

\[ 1 - (\sqrt{3} - 1) \cdot 1 = 2 - \sqrt{3} \]
\[ (\sqrt{3} - 1) - (2 - \sqrt{3}) \cdot 2 = -5 + 3\sqrt{3} \]
\[ (2 - \sqrt{3}) - (\sqrt{3} - 1) \cdot 1 = 7 - 4\sqrt{3} \]
\[ (\sqrt{3} - 1) - (2 - \sqrt{3}) \cdot 2 = -19 + 11\sqrt{3} \]
\[ (\sqrt{3} - 1) - (2 - \sqrt{3}) \cdot 2 = -71 + 41\sqrt{3} \]

In both cases, the fact that the algorithm terminates at zero attests to commensurability.

They must say this for field credit.

\[ \text{Irrationality of } \sqrt{3} \text{ is equivalent to the fact that the algorithm does not terminate. In fact, the quotients (the column circled in red) go } 1, 1, 2, 1, 2, 1, 2, \ldots \text{ alternating 1 & 2 ad infinitum.} \]

Not needed for credit.

\[ \text{What this really says—and the way you can prove it—is that } \]
\[ \sqrt{3} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \ldots}}}} \]

which can be verified by showing \( \frac{1}{x - 1} \) is a solution to

\[ \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \ldots}}}} = x \]

\[ x = 1 + \frac{1}{1 + \frac{1}{2 + (x - 1)}} \]
2. Aristotle summarizes an argument by contradiction that \( \sqrt{2} \) is the diagonal of a square of side 1 is irrational (i.e., the diagonal and the side are incommensurate). The argument assumes \( \sqrt{2} \) is rational, and indicates that this will force some integer to be both even and odd ("odd numbers come out equal to evens"). Namely, in modern symbolism, if \( \sqrt{2} = \frac{p}{q} \) with \( p \) and \( q \) relatively prime integers, then \( 2q^2 = p^2 \). By assumption, at most one of \( p, q \) is even; the equation forces \( p \) to be even, since 2 divides \( p^2 \).

Set \( p = 2p_0 \) for \( p_0 \in \mathbb{Z} \). Then \( q^2 = 2p_0^2 \). Forcing \( q \) to be even, whereas our assumption has already implied \( q \) is odd. A contradiction results, since \( q \) cannot be both even and odd.