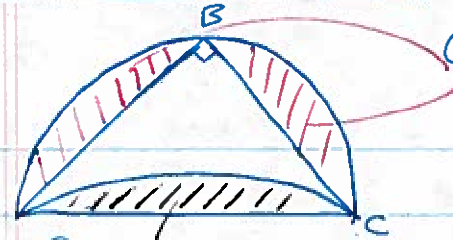
Math 3010 Greek Geometry HW

(1) (a)

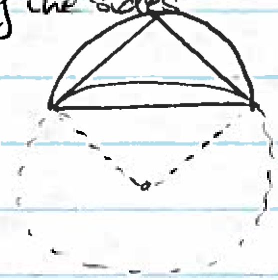


① the segments of the (drawn) semicircle cut off by the sides

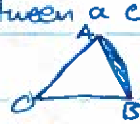
Items ①②③ are what the answer requires

② the "segment of a circle similar to those cut off by the sides"

this circle is not drawn in the picture; it is the dotted circle in this diagram:



③ By a segment of a circle, Simplicius means a region between a chord ($\overline{AB}$) of a circle and the portion of the circumference ($\widehat{AB}$) it cuts off (ie, what is shaded in this picture)



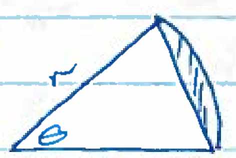
(b) All three of these segments are the segments of quarter-circles (cutting off the same angle is what it means to be "similar").

(Sorry for the mess. The 4 boxed numbers are what the answer requires)

(c) • If the large semi-circle has radius 1, then the triangle ABC has area 1 $\left(= \frac{1}{2}\frac{2}{\sqrt{2}}\cdot\frac{2}{\sqrt{2}}\right)$; Hippocrates's argument tells us $\triangle ABC$ has the same area as the lune

which is therefore also $\boxed{1}$

• The area of a segment of angle θ in a circle of radius r is



$$\frac{\theta}{2\pi}\cdot\pi r^2 - \frac{1}{2}r\cdot r\sin\theta = \frac{r^2}{2}(\theta - \sin\theta)$$

– The segments cut off by ~~between~~ AB and BC then have area ($r=1$, $\theta = \pi/2$)

$$\frac{1}{2}\left(\pi/2 - \sin(\pi/2)\right) = \boxed{\frac{\pi}{4} - \frac{1}{2}}$$

– The segment cut off by AC has area ($r = 2/\sqrt{2}$, $\theta = \pi/2$) $\boxed{\frac{\pi}{2} - 1}$

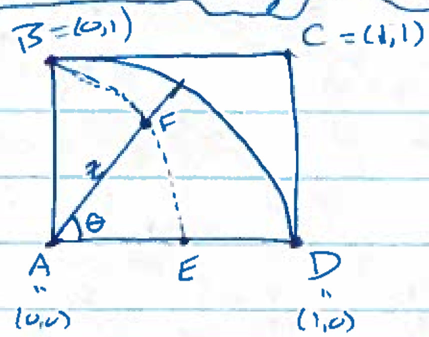
– The area of

the region obtained by deleting the segment AC from $\triangle ABC$ is

$$1 - \left(\frac{\pi}{2} - 1\right) = \boxed{2 - \pi/2}$$

(Note that the 4 areas add up to $\pi/2$ = area semi-circle).

3010 Greek Geometry HW (ct'd)

(2) 

The defining property of the quadratix (the dotted curve) implies that $z = \dfrac{\theta}{\frac{\pi}{2}\cdot \sin\theta}$

(a) When the radius of the circle has rotated half-way from AB to AD it is a segment of line AC; at this time the segment BC has moved half-way down the square, resulting in a segment that intersects AC at the ~~midpoint~~ center of the square.

(b) Take $\theta = \pi/6$. Then $z = \dfrac{\pi/6}{\frac{\pi}{2}\cdot\frac{1}{2}} = \dfrac{2}{3}$, so the corresponding point (F in the figure) on the quadratix is $(z\cos\theta, z\sin\theta) = \left(\frac{2}{3}\cdot\frac{\sqrt{3}}{2}, \frac{2}{3}\cdot\frac{1}{2}\right) = \boxed{\left(\frac{\sqrt{3}}{3}, \frac{1}{3}\right)}$

(c) We want to know z when $\theta = 0$. The expression $\dfrac{\theta}{\frac{\pi}{2}\sin\theta}$ is not defined at zero, but we can compute the length AE as $\boxed{\lim_{\theta\to 0} \dfrac{\theta}{\frac{\pi}{2}\sin\theta} = \dfrac{2}{\pi}}$