**Math 3010 Homework, Due 12 February, 2016**
**Early Greek Geometry**

(1) Our most reliable source for Hippocrates's work on the "quadrature of lunes" (a "lune" is a shape that looks like a crescent moon; "quadrature of a lune" is the problem of finding a rectilinear shape with the same area as the lune) is a 6th century CE [sic!] commentary by Simplicius, supposedly quoting from Eudemus's lost (4th c. BCE) *History of Geometry*. Read the translated text[1] (with diagram on the facing page) of Simplicius, on page 2 of this homework assignment, and answer the following questions:
   (a) (2pts) What does Simplicius mean by a "segment" of a circle? Redraw the diagram and clearly label the various "segments" Simplicius refers to.
   (b) (1pt) Why is the "segment" circumscribed about the base of the right triangle similar to the segments cut off by its sides?
   (c) (3pts) If the large semicircle is taken to have radius 1, what is the area of the lune? What are the areas of each of the four non-overlapping regions in the diagram?

(2) This problem concerns the *quadratix*, likely introduced by Hippias in the late 5th century BCE. Before doing this problem, you should review the construction we gave in class. Now, for definiteness, suppose we have constructed a quadratix using the square with vertices

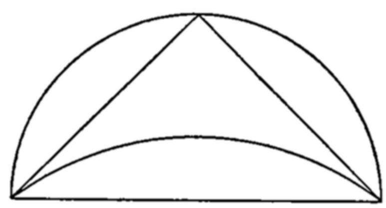
$$A = (0,0), B = (0,1), C = (1,1), D = (1,0)$$

and its inscribed quarter-circle centered at $A$ (that is, the quadratix starts at the point $B$ and terminates at a point–let us call it $E$–along the $AD$-axis).
   (a) (1pt) Explain geometrically why the quadratix passes through the midpoint of the square.
   (b) (2pts) Find (exactly) the coordinates of the point of intersection of the quadratix with the line emanating from $A$ at an angle of 30 degrees above the $AD$-axis.
   (c) (3pts) Find the coordinates of the point of intersection of the quadratix with the $AD$-axis.

[1] *Greek Mathematical Works, Volume I: Thales to Euclid*, translated by Ivor Thomas, Loeb Classical Library (vol. 335), Harvard University Press, pages 238-239.

" Καὶ οἱ τῶν μηνίσκων δὲ τετραγωνισμοὶ δόξαντες εἶναι τῶν οὐκ ἐπιπολαίων διαγραμμάτων διὰ τὴν οἰκειότητα τὴν πρὸς τὸν κύκλον ὑφ' Ἱπποκράτους ἐγράφησάν τε πρώτου καὶ κατὰ τρόπον ἔδοξαν ἀποδοθῆναι· διόπερ ἐπὶ πλέον ἁψώμεθά τε καὶ διέλθωμεν. ἀρχὴν μὲν οὖν ἐποιήσατο καὶ πρῶτον ἔθετο τῶν πρὸς αὐτοὺς χρησίμων, ὅτι τὸν αὐτὸν λόγον ἔχει τά τε ὅμοια τῶν κύκλων τμήματα πρὸς ἄλληλα καὶ αἱ βάσεις αὐτῶν δυνάμει. τοῦτο δὲ ἐδείκνυεν ἐκ τοῦ τὰς διαμέτρους δεῖξαι τὸν αὐτὸν λόγον ἐχούσας δυνάμει τοῖς κύκλοις.

" Δειχθέντος δὲ αὐτῷ τούτου πρῶτον μὲν ἔγραφε μηνίσκου τὴν ἐκτὸς περιφέρειαν ἔχοντος ἡμικυκλίου



τίνα τρόπον γένοιτο ἂν τετραγωνισμός. ἀπεδίδου δὲ τοῦτο περὶ τρίγωνον ὀρθογώνιόν τε καὶ ἰσοσκελὲς ἡμικύκλιον περιγράψας καὶ περὶ τὴν βάσιν τμῆμα κύκλου τοῖς ὑπὸ τῶν ἐπιζευχθεισῶν ἀφαιρουμένοις ὅμοιον. ὄντος δὲ τοῦ περὶ τὴν βάσιν τμήματος ἴσου τοῖς περὶ τὰς ἑτέρας ἀμφοτέροις, καὶ κοινοῦ προστεθέντος τοῦ μέρους τοῦ τριγώνου τοῦ ὑπὲρ τὸ τμῆμα τὸ περὶ τὴν βάσιν, ἴσος ἔσται ὁ μηνίσκος τῷ τριγώνῳ. ἴσος οὖν ὁ μηνίσκος τῷ τριγώνῳ δειχθεὶς τετραγωνίζοιτο ἄν. οὕτως μὲν





" The quadratures of lunes, which seemed to belong to an uncommon class of propositions by reason of the close relationship to the circle, were first investigated by Hippocrates, and seemed to be set out in correct form; therefore we shall deal with them at length and go through them. He made his starting-point, and set out as the first of the theorems useful to his purpose, that similar segments of circles have the same ratios as the squares on their bases.[a] And this he proved by showing that the squares on the diameters have the same ratios as the circles.[b]

" Having first shown this he described in what way it was possible to square a lune whose outer circumference was a semicircle. He did this by circumscribing about a right-angled isosceles triangle a semicircle and about the base a segment of a circle similar to those cut off by the sides.[c] Since the segment about the base is equal to the sum of those about the sides, it follows that when the part of the triangle above the segment about the base is added to both the lune will be equal to the triangle. Therefore the lune, having been proved equal to the triangle, can be squared. In this way, taking

[a] Lit. " as the bases in square."

[b] This is Eucl. xii. 2 (see *infra*, pp. 458-465). Euclid proves it by a method of exhaustion, based on a lemma or its equivalent which, on the evidence of Archimedes himself, can safely be attributed to Eudoxus. We are not told how Hippocrates effected the proof.

[c] As Simplicius notes, this is the problem of Eucl. iii. 33 and involves the knowledge that similar segments contain equal angles.