Math 3010 Homework due March 11: Conics

Consider the parabola $\mathcal{P}$ whose equation in the plane (with the usual x and y coordinates) is given by $y = x^2$. In this homework you will rederive using analytic geometry some of the properties of the parabola that we established in class via more classical geometric arguments.

(1) (2pts) Determine the focus and directrix of $\mathcal{P}$.

(2) (2pts) Show using analytic geometry that for any point P on $\mathcal{P}$, whose closest point on the directrix we denote by P', the tangent to $\mathcal{P}$ at P is the perpendicular bisector of the segment $\overline{P'F}$ connecting P' to the focus F.

(3) (2pts) Let $P = (a, a^2)$ and $Q = (b, b^2)$ be any two distinct points on $\mathcal{P}$. Compute the coordinates of the third vertex R of the Archimedes triangle whose base is $\overline{PQ}$ (that is, compute the point of intersection of the tangents to $\mathcal{P}$ at P and Q).

(4) (4pts) Use calculus to compute the area (in terms of a and b) of the parabolic segment bounded by $\overline{PQ}$.

(5) (2pts) Compute the area of triangle PQR; in combination with the previous question, you will now have reproved Archimedes' theorem on quadrature of the parabola $\mathcal{P}$.