29. For the function f graphed in Figure 11, find the indicated limit or function value, or state that it does not exist.

(a) $\lim_{x\to -3} f(x)$ (b) $f(-3)$ (c) $f(-1)$

(d) $\lim_{x\to -1} f(x)$ (e) $f(1)$ (f) $\lim_{x\to 1} f(x)$

(g) $\lim_{x\to 1} f(x)$ (h) $\lim_{x\to 1^+} f(x)$ (i) $\lim_{x\to -1^+} f(x)$

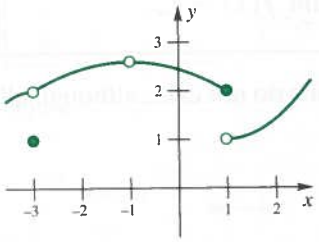

Figure 11

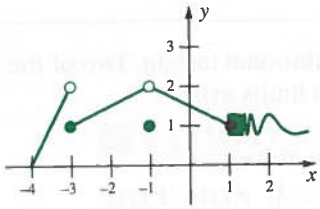

Figure 12

30. Follow the directions of Problem 29 for the function f graphed in Figure 12.

31. For the function f graphed in Figure 13, find the indicated limit or function value, or state that it does not exist.

(a) $f(-3)$ (b) $f(3)$ (c) $\lim_{x\to -3^-} f(x)$

(d) $\lim_{x\to -3^+} f(x)$ (e) $\lim_{x\to -3} f(x)$ (f) $\lim_{x\to 3^+} f(x)$

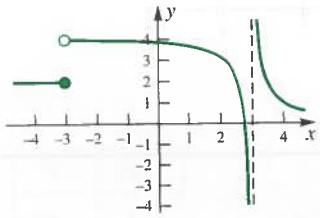

Figure 13

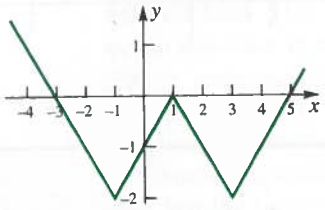

Figure 14

32. For the function f graphed in Figure 14, find the indicated limit or function value, or state that it does not exist.

(a) $\lim_{x\to -1^-} f(x)$ (b) $\lim_{x\to -1^+} f(x)$ (c) $\lim_{x\to -1} f(x)$

(d) $f(-1)$ (e) $\lim_{x\to 1} f(x)$ (f) $f(1)$

33. Sketch the graph of

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } 0 \le x < 1 \\ 1 + x & \text{if } x \ge 1 \end{cases}$$

Then find each of the following or state that it does not exist.

(a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to 1} f(x)$

(c) $f(1)$ (d) $\lim_{x\to 1^+} f(x)$

34. Sketch the graph of

$$g(x) = \begin{cases} -x + 1 & \text{if } x < 1 \\ x - 1 & \text{if } 1 < x < 2 \\ 5 - x^2 & \text{if } x \ge 2 \end{cases}$$

Then find each of the following or state that it does not exist.

(a) $\lim_{x\to 1} g(x)$ (b) $g(1)$

(c) $\lim_{x\to 2} g(x)$ (d) $\lim_{x\to 2^+} g(x)$

35. Sketch the graph of $f(x) = x - [x]$; then find each of the following or state that it does not exist.

(a) $f(0)$ (b) $\lim_{x\to 0} f(x)$

(c) $\lim_{x\to 0^-} f(x)$ (d) $\lim_{x\to 1/2} f(x)$

36. Follow the directions of Problem 35 for $f(x) = x/|x|$.

37. Find $\lim_{x\to 1}(x^2 - 1)/|x - 1|$ or state that it does not ex

38. Evaluate $\lim_{x\to 0}\left(\sqrt{x+2} - \sqrt{2}\right)/x$. *Hint:* Rationalize numerator by multiplying the numerator and denominator $\sqrt{x+2} + \sqrt{2}$.

39. Let

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ -x & \text{if } x \text{ is irrational} \end{cases}$$

Find each value, if possible.

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 0} f(x)$

40. Sketch, as best you can, the graph of a function f t satisfies all the following conditions.

(a) Its domain is the interval $[0, 4]$.

(b) $f(0) = f(1) = f(2) = f(3) = f(4) = 1$

(c) $\lim_{x\to 1} f(x) = 2$ (d) $\lim_{x\to 2} f(x) = 1$

(e) $\lim_{x\to 3^-} f(x) = 2$ (f) $\lim_{x\to 3^+} f(x) = 1$

41. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ x^4 & \text{if } x \text{ is irrational} \end{cases}$$

For what values of a does $\lim_{x\to a} f(x)$ exist?

42. The function $f(x) = x^2$ had been carefully graphed, during the night a mysterious visitor changed the values of f million different places. Does this affect the value of $\lim_{x\to a} f(x$ any a? Explain.

43. Find each of the following limits or state that it does exist.

(a) $\lim_{x\to 1} \dfrac{|x-1|}{x-1}$ (b) $\lim_{x\to 1^-} \dfrac{|x-1|}{x-1}$

(c) $\lim_{x\to 1^-} \dfrac{x^2 - |x-1| - 1}{|x-1|}$ (d) $\lim_{x\to 1^-}\left[\dfrac{1}{x-1} - \dfrac{1}{|x-}\right.$

44. Find each of the following limits or state that it does exist.

(a) $\lim_{x\to 1^+} \sqrt{x - [x]}$ (b) $\lim_{x\to 0^+} [1/x]$

(c) $\lim_{x\to 0^+} x(-1)^{[1/x]}$ (d) $\lim_{x\to 0^+} [x](-1)^{[1/x]}$

45. Find each of the following limits or state that it does exist.

(a) $\lim_{x\to 0^+} x[1/x]$ (b) $\lim_{x\to 0^+} x^2[1/x]$

(c) $\lim_{x\to 3^-} ([x] + [-x])$ (d) $\lim_{x\to 3^+} ([x] + [-x])$

46. Find each of the following limits or state that it does exist.

(a) $\lim_{x\to 3} [x]/x$ (b) $\lim_{x\to 0^+} [x]/x$

(c) $\lim_{x\to 1.8} [x]$ (d) $\lim_{x\to 1.8} [x]/x$

CAS *Many software packages have programs for calculating its, although you should be warned that they are not infallibl develop confidence in your program, use it to recalculate son the limits in Problems 1–28. Then for each of the following, the limit or state that it does not exist.*

47. $\lim_{x\to 0} \sqrt{x}$ **48.** $\lim_{x\to 0^+} x^x$

49. $\lim_{x\to 0} \sqrt{|x|}$ **50.** $\lim_{x\to 0} |x|^x$