Math 1210-021, Midterm 2 Review Problems

The second midterm will focus on the material from sections 3.1-4.2.

1. Identify the critical points and find the extreme values of the following functions on the following intervals:

   (a) $f(x) = x^3 - 3x^2 - 15x + 1$ on $[0, 4]$

   (b) $f(x) = -2\cos(x) - x^2$ on $[-\frac{\pi}{2}, \pi]$.

2. Let $f(x) = \frac{1+x}{\sqrt{x}}$. Identify the following properties of $f(x)$ and then sketch a graph of the function:

   - domain
   - critical points and local extrema (and whether they are maxima or minima)
   - inflection points
   - asymptotes

3. Let $f(x) = x^7 + 2x^5 + x - 8$.

   (a) Use the Intermediate Value Theorem to show $f(x)$ has at least one real root.

   (b) Use the Mean Value Theorem to show $f(x)$ has at most one real root, hence has exactly one real root.

4. Let $f(x) = \frac{1}{9}x^4 - 2x^2 + 2$. Identify where $f(x)$ is increasing/decreasing and concave up/concave down. How many zeroes does $f(x)$ have? Combine this with whatever other information you think is appropriate to sketch a graph of $f(x)$.

5. A motorist is in the desert in a car. The nearest road, which is perfectly straight, is $4\sqrt{2}$ miles away, and then ten miles along the road there is a town. The motorist wishes to reach the town as quickly as possible. If he/she can drive 45mph on-road and 15mph off-road, what is the quickest route to the town? (You may assume this route consists of a straight path through the desert followed by a straight path on the road.)

6. Carry out two iterations of Newton's method on the function $f(x) = x^2 - 5$ with the initial estimate $x_0 = 2$. Draw a picture (i.e., starting from a graph of $f(x)$) illustrating the algorithm in this example.

7. Compute the following antiderivatives:

   (a) $\displaystyle\int (x^{7/3} - 9x^5 + 2)dx$

   (b) $\displaystyle\int x\sqrt{x^2+1}dx$. Also find the particular antiderivative $F(x)$ such that $F(\sqrt{15}) = 1$.

   (c) $\displaystyle\int (x\tan(x))^3(x\sec^2(x) + \tan(x))dx$

8. Find a function $y = y(x)$ satisfying $\frac{dy}{dx} = \frac{\sqrt[3]{x}}{y}$ and $y(8) = 0$. What is the domain of the function y you have found? Where is it differentiable? In particular, does it satisfy the given differential equation on its entire domain?

9. Use the definition of the definite integral via Riemann sums to compute $\displaystyle\int_1^3 x^2 dx$. Explain (a picture might help) the geometric meaning of this quantity.