Math 1210-021, Midterm 2, 11/6/2017

Your course-specific ID #:____________________________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. Put a $\boxed{\text{box}}$ around your final answers. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed. Good luck!

Question	Points	Score
1	9	
2	7	
3	8	
4	7	
5	6	
6	9	
7	8	
8	9	
Total:	63	

1. Determine the point(s), *if any*, at which the following functions attain global maxima and global minima on the indicated intervals. Be sure to explain your answer carefully.

   (a) (4 points) $f(x) = \dfrac{x}{x^2+1}$ on $(-\infty, \infty)$.

   (b) (5 points) $f(x) = \sin^2(x) + \cos(x)$ on $[0, 3]$.

2. (a) (1 point) Give the *definition* (as given in class) of what it means for a function to be concave up; and what it means to be concave down.

(b) (3 points) Where is the function $g(x) = x^4 + 2x^3 - 6x^2 + 11x + 13$ concave up, and where is it concave down?

(c) (3 points) For which positive integers n does the function $f(x) = x^n$ have an inflection point?

3. Consider the function $f(x) = x^3 - 6x^2 + 9x + 3$.

   (a) (3 points) Identify the critical points and inflection points of $f(x)$.

(b) (5 points) Using the previous part, and any other information you deem relevant, sketch a graph of the function $f(x)$.

4. (7 points) What are the dimensions and area of the rectangle of maximum area that can be inscribed in a circle of radius r?

5. (a) (4 points) Describe geometrically the algorithm in Newton's method, and then derive (starting from your description) the recursive formula giving the $(n+1)^{st}$ approximation in terms of the n^{th} approximation.

(b) (2 points) Let $f(x) = x^3 - x - 1$. Consider the first estimate $x_1 = 1$ of a zero of $f(x)$. What is the next approximation x_2 obtained from Newton's method?

6. Compute the following antiderivatives:

(a) (3 points) $\int z^{-\frac{2}{3}} + z^{-2} + z^{10} dz$

(b) (3 points) $\int \sin(x)\cos(x)^{\frac{1}{3}} dx$.

(c) (3 points) The particular antiderivative F of $\int \sec^2(t)dt$ such that $F(\frac{\pi}{4}) = -1$.

7. (8 points) Compute the definite integral $\int_1^3 (x - x^2)dx$ using *right endpoint* Riemann sums.

8. Determine whether the following statements are true or false; **to receive credit, write out the full word "true" or "false" rather than "T" or "F".**

   (a) ($1\frac{1}{2}$ points) If $f(x)$ is any function on the interval $[0, 1]$ such that $f(0) = -1$ and $f(1) = 1$, then the bisection method will yield a sequence of real numbers whose limit is a zero of $f(x)$.

   (b) ($1\frac{1}{2}$ points) There exists a function with domain all real numbers that is both (i) increasing everywhere; and (ii) concave down everywhere.

   (c) ($1\frac{1}{2}$ points) A cubic polynomial must have at least one real root.

   (d) ($1\frac{1}{2}$ points) Every continuous function on the interval $[3, 5]$ is Riemann-integrable.

   (e) ($1\frac{1}{2}$ points) Every rational function that is not a polynomial has at least one vertical, horizontal, or slant asymptote.

   (f) ($1\frac{1}{2}$ points) $\sin(x) \le x^2 + \frac{x}{2}$ for all real numbers x.