(1) (a) \( \lim_{x \to 0} \frac{\tan(\frac{x}{2})}{2x} = \lim_{x \to 0} \frac{1}{6} \frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})} = \frac{1}{6} \)

(b) \( \lim_{x \to 0^-} \cos(1/x) \) DNE (oscillates wildly).

(c) \( \lim_{x \to 1} x \lfloor x \rfloor \) DNE (the left-hand limit is 0, while the right-hand limit is 1).

(d) \( \lim_{x \to -\infty} \frac{3x^3 - 7x + 12}{5x^3 + 9x^2} = \frac{3}{5} \)

(e) \( \lim_{x \to \infty} \frac{\tan(x)}{x} \) DNE (on every interval \((\frac{\pi}{2} + n\pi, \frac{\pi}{2} + (n+1)\pi)\), the function hits all real numbers)

(f) \( \lim_{x \to 1} \frac{(x-1)^3}{x - \pi} \) DNE (\(-\infty\) from the left and \(+\infty\) from the right)

(g) \( \lim_{x \to \pi} \frac{\cos(x)}{x - \pi} = \lim_{y \to 0} \frac{\cos(y + \frac{\pi}{2})}{y} = \lim_{y \to 0} \frac{-\sin(y)}{y} = -1 \)

(2) (a) \( f'(a) = \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} = \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{2\sqrt{a}} \)

(b) \( f'(a) = \lim_{x \to a} \frac{1/x - 1/a}{x - a} = \lim_{x \to a} \frac{1}{ax} = -\frac{1}{a^2} \)

(3) (a) Since \( f'(a) = 3a^2 - 3 \), the equation of the tangent line at \( x = a \) is \( y - (a^3 - 3a - 1) = (3a^2 - 3)(x - a) \)

(b) The tangent line is horizontal when \( 3a^2 - 3 = 0 \), i.e. when \( a = 1 \) or \( a = -1 \).

(c) Intermediate Value Theorem: Let \( f \) be a continuous function on a closed interval \([a, b]\), and let \( \alpha \) be any real number strictly between \( f(a) \) and \( f(b) \). Then there exists an \( c \in (a, b) \) such that \( f(c) = \alpha \).

To show that \( f(x) = 0 \) has three distinct real solutions, note that \( f \) is everywhere continuous, so the IVT applies on any closed interval. Since \( f(-2) < 0, f(-1) > 0, f(1) < 0, \) and \( f(2) > 0 \), the IVT shows that \( f(c) = 0 \) for some \( c \in (-2, -1), \) some \( c \in (-1, 1), \) and some \( c \in (1, 2). \)

(4) (a) \( \frac{dy}{dx} = \frac{(x^2 + 1)(x \cos(x) + \sin(x)) - x \sin(x)2x}{(x^2 + 1)^2} = \frac{(x^3 + x) \cos(x) + (-x^2 + 1) \sin(x)}{(x^2 + 1)^2} \)

(b) \( g'(t) = -t^{-\frac{3}{2}} \csc^2(t) - \frac{1}{4} t^{-\frac{5}{2}} \cot(t) \)

(c) The two points in question are \((\frac{3}{5}, \frac{1}{5})\) and \((-\frac{3}{5}, \frac{1}{5})\). The general formula for the derivative is \( 2x + 32y \frac{dy}{dx} = 0 \), i.e. \( \frac{dy}{dx} = -\frac{x}{16y} \). Subbing in, we find that the slopes of the tangent lines at \((\frac{3}{5}, \frac{1}{5})\) and \((-\frac{3}{5}, \frac{1}{5})\), respectively, are \(-\frac{3}{16} \) and \( \frac{3}{16} \).

(5) (a) Recall that \( f'(x) = \sec^2(x) \). Starting from \( \frac{\pi}{6} \), the approximation is \( f(\frac{\pi}{6}) \approx f(\frac{\pi}{6}) + f'(\frac{\pi}{6})(\frac{\pi}{6} - \frac{\pi}{6}) = \frac{1}{\sqrt{3}} + \frac{4 \pi}{30} = \frac{1}{\sqrt{3}} + \frac{2\pi}{45} \)

(b) By inspection of the graph of the tangent function, for any \( x \in (0, \frac{\pi}{6}) \), the tangent line to \( f \) at \( x \) lies below the graph of \( y = f(x) \) (as long as one stays in the interval \((0, \frac{\pi}{6}) \)). Since we are approximating \( f(\frac{\pi}{6}) \) by taking the values at \( \frac{\pi}{5} \) of the tangent lines to \( y = f(x) \) at \( \frac{\pi}{5} \) and \( \frac{\pi}{10} \), in both cases we get \( \approx \) underestimates.
Let $a(t)$ be the distance traveled by person A, let $b(t)$ be the distance traveled by person B, and let $d(t)$ be the distance between them. By the law of cosines,

$$d(t)^2 = a(t)^2 + b(t)^2 - 2a(t)b(t)\cos(\frac{3\pi}{4}) = a(t)^2 + b(t)^2 + \sqrt{2}a(t)b(t).$$

(If you forget the law of cosines, just drop the perpendicular from person B to the axis where person A is traveling, and apply the Pythagorean theorem to that right triangle.) Differentiating, we get

$$2d(t)d'(t) = 2a(t)a'(t) + 2b(t)b'(t) + \sqrt{2}(a(t)b'(t) + a'(t)b(t)).$$

When $a = 1$ and $b = \sqrt{2}$, the first formula shows that $d = \sqrt{5}$, so when $a' = 1$ and $b' = 3\sqrt{2}$, we can plug everything in to the derivative formula to get (at this moment $t_0$ of interest, and where we’ve already divided by 2)

$$\sqrt{5}d'(t_0) = 1 + \sqrt{2} \cdot 3\sqrt{2} + \frac{\sqrt{2}}{2}(3\sqrt{2} + \sqrt{2}),$$

i.e. $d'(t_0) = \frac{11}{\sqrt{5}}$ leagues/hour.

(7) In order from top to bottom, the correct labeling is $\boxed{D, B, A, E, C}$.