Your course-specific ID #: _______________________

Instructions: Please show all of your work as partial credit will be given where appropriate, and there may be no credit given for problems where there is no work shown. Put a box around your final answers. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td>71</td>
<td></td>
</tr>
</tbody>
</table>
1. Evaluate the following limits. Recall our conventions from class: infinite limits should be recorded as such, and other cases where there is neither a finite nor an infinite limit should be recorded as “DNE.”
   
   (a) (2 points) \( \lim_{x \to 0} \frac{\tan\left(\frac{x}{3}\right)}{2x} \)

   (b) (2 points) \( \lim_{x \to 0^-} \cos\left(\frac{1}{x}\right) \)

   (c) (2 points) \( \lim_{x \to 1} x \lfloor x \rfloor \)

   (d) (2 points) \( \lim_{x \to -\infty} \frac{3x^3 - 7x + 12}{5x^3 + 9x^2} \)
(e) (2 points) \[ \lim_{x \to \infty} \frac{\tan(x)}{x} \]

(f) (2 points) \[ \lim_{x \to 1} \frac{1}{(x - 1)^3} \]

(g) (2 points) \[ \lim_{x \to \frac{\pi}{2}} \frac{\cos(x)}{x - \frac{\pi}{2}} \]
2. Evaluate the derivatives $f'(x)$ of the following functions using the limit definition of the derivative. (Other methods will receive no credit, but you may use any correct form of the limit definition.)

(a) (4 points) $f(x) = \sqrt{x}$

(b) (4 points) $f(x) = \frac{1}{x}$
3. Let \( f(x) = x^3 - 3x - 1 \)
   
   (a) (3 points) For any real number \( a \), compute the equation of the tangent line to \( y = f(x) \) at the point where \( x = a \).

   

   (b) (2 points) Where (for what values of \( a \)) is the tangent line horizontal?

   

   (c) (5 points) State the Intermediate Value Theorem precisely, and use it to show that the equation \( f(x) = 0 \) has three distinct real solutions.
4. Calculate the following derivatives.
   
   (a) (4 points) \( \frac{dy}{dx} \) where \( y = \frac{x \sin(x)}{x^2 + 1} \)

   (b) (4 points) \( g'(t) \) where \( g(t) = t^{-\frac{1}{2}} \cot(t) \)
(c) (5 points) The slope of the tangent line to the ellipse \( x^2 + 16y^2 = 1 \) at each point where \( y = \frac{1}{5} \).

5. Let \( f(x) = \tan(x) \).

(a) (6 points) Estimate \( f(\frac{\pi}{3}) \) using two different linear approximations, one starting from the value \( f(\frac{\pi}{6}) \), and one starting from the value \( f(\frac{\pi}{4}) \). (Your answer will involve \( \pi \): do not use a decimal approximation.)
(b) (2 points) Determine whether each of your estimates is an over-estimate or an under-estimate. Be sure to explain your reasoning.

6. (8 points) Two people depart from the same point. Person A goes due east, and Person B goes due northwest. At the time when Person A has traveled 1 league (a unit of distance), Person B has traveled $\sqrt{2}$ leagues. Suppose furthermore that at this moment Person A is traveling at 1 league/hour, while Person B is traveling at $3\sqrt{2}$ leagues/hour. At what rate is the distance between the two people increasing (at this particular moment)?
7. (10 points) Match each of the following graphs in the left-hand column to its derivative in the right-hand column. Record your answer by labeling the left-hand graphs with the appropriate boxed letter next to the corresponding right-hand graph.