Math 1210-021, Midterm 1, 10/2/2017

Your course-specific ID #:_____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. Put a $\boxed{\text{box}}$ around your final answers. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed.

Question	Points	Score
1	14	
2	8	
3	10	
4	13	
5	8	
6	8	
7	10	
Total:	71	

1. Evaluate the following limits. Recall our conventions from class: infinite limits should be recorded as such, and other cases where there is neither a finite nor an infinite limit should be recorded as "DNE."

   (a) (2 points) $\lim_{x \to 0} \frac{\tan(\frac{x}{3})}{2x}$

   (b) (2 points) $\lim_{x \to 0^-} \cos(1/x)$

   (c) (2 points) $\lim_{x \to 1} x\lfloor x \rfloor$

   (d) (2 points) $\lim_{x \to -\infty} \frac{3x^3 - 7x + 12}{5x^3 + 9x^2}$

(e) (2 points) $\lim_{x\to\infty} \frac{\tan(x)}{x}$

(f) (2 points) $\lim_{x\to 1} \frac{1}{(x-1)^3}$

(g) (2 points) $\lim_{x\to \frac{\pi}{2}} \frac{\cos(x)}{x-\frac{\pi}{2}}$

2. Evaluate the derivatives $f'(x)$ of the following functions *using the limit definition of the derivative.* (Other methods will receive no credit, but you may use any correct form of the limit definition.)

   (a) (4 points) $f(x) = \sqrt{x}$

   (b) (4 points) $f(x) = \frac{1}{x}$

3. Let $f(x) = x^3 - 3x - 1$

(a) (3 points) For any real number a, compute the equation of the tangent line to $y = f(x)$ at the point where $x = a$.

(b) (2 points) Where (for what values of a) is the tangent line horizontal?

(c) (5 points) State the Intermediate Value Theorem precisely, and use it to show that the equation $f(x) = 0$ has three distinct real solutions.

4. Calculate the following derivatives.

(a) (4 points) $\frac{dy}{dx}$ where $y = \frac{x\sin(x)}{x^2+1}$

(b) (4 points) $g'(t)$ where $g(t) = t^{-\frac{1}{4}}\cot(t)$

(c) (5 points) The slope of the tangent line to the ellipse $x^2 + 16y^2 = 1$ at *each* point where $y = \frac{1}{5}$.

5. Let $f(x) = \tan(x)$.

   (a) (6 points) Estimate $f(\frac{\pi}{5})$ using two different linear approximations, one starting from the value $f(\frac{\pi}{6})$, and one starting from the value $f(\frac{\pi}{4})$. (Your answer will involve π: do not use a decimal approximation.)

(b) (2 points) Determine whether each of your estimates is an over-estimate or an under-estimate. Be sure to explain your reasoning.

6. (8 points) Two people depart from the same point. Person A goes due east, and Person B goes due northwest. At the time when Person A has traveled 1 league (a unit of distance), Person B has traveled $\sqrt{2}$ leagues. Suppose furthermore that at this moment Person A is traveling at 1 league/hour, while Person B is traveling at $3\sqrt{2}$ leagues/hour. At what rate is the distance between the two people increasing (at this particular moment)?

7. (10 points) Match each of the following graphs in the left-hand column to its derivative in the right-hand column. Record your answer by labeling the left-hand graphs with the appropriate boxed letter next to the corresponding right-hand graph.